Differential Equations with Linear Algebra (M427J)

Some answers to practice problems # 4

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§2.5 # 2. There are two cases: $\alpha = -2$ and $\alpha \neq -2$.

Case 1. If $\alpha = -2$, then $\Psi(t) = \dfrac{1}{6}\, t^3\, e^{-2t}$. In this case, we use $s = 2$ and $n = 1$.

Case 2. If $\alpha \neq -2$, then we need to use $s = 0$ and $n = 1$. We find that

$$\Psi(t) = \left(\frac{-2}{(\alpha+2)^3} + \frac{1}{(\alpha+2)^2}\, t \right) e^{\alpha t} = \frac{(\alpha t + 2t - 2)\, e^{\alpha t}}{(\alpha+2)^3}.$$

§2.5 # 4. $\Psi(t) = t^2 - t.$

§2.5 # 6. $\Psi(t) = \dfrac{\left(273 - 1672t + 3872t^2\right) e^{7t}}{340736}.$

§2.5 # 7. The characteristic equation is $r^2 + 4 = 0$. We have two complex roots: $2i$ and $-2i$. The right hand side of the non-homogeneous equation is of the form

$$g(t) = t\, \sin(2t) = P(t)\, e^{\alpha t} \cos(\beta t) + Q(t)\, e^{\alpha t} \sin(\beta t).$$

We will use $\alpha = 0$ and $\beta = 2$, and we have that $\alpha + i\beta$ is a single root, so we will use $s = 1$. Also, $P(t)$ is a polynomial of degree 0 and $Q(t)$ is a polynomial of degree 1. We will use $n = 1$.

The considerations above imply that the form of the function $\Psi(t)$ is

$$\begin{aligned} \Psi(t) &= t^s \left[(A_0 + A_1\, t)\, e^{\alpha t} \cos(\beta t) + (B_0 + B_1\, t)\, e^{\alpha t} \sin(\beta t) \right] \\ &= \left(A_0\, t + A_1\, t^2\right) \cos(2t) + \left(B_0\, t + B_1\, t^2\right) \sin(2t). \end{aligned}$$

Computing the linear operator, we find that

$$L[\Psi] = \Psi''(t) + 4\Psi(t) = 2\,(A_1 + 2B_0) \cos(2t) + (8B_1)\, t \cos(2t) + 2\,(B_1 - 2A_0) \sin(2t) + (-8A_1)\, t \sin(2t).$$

Matching coefficients, we get the equations

$$A_1 + 2B_0 = 0, \qquad 8B_1 = 0, \qquad B_1 - 2A_0 = 0, \qquad -8A_1 = 1.$$

Solving these equations we find that

$$A_0 = 0, \qquad A_1 = -\frac{1}{8}, \qquad B_0 = \frac{1}{16}, \qquad B_1 = 0.$$

We conclude that $\Psi(t) = \dfrac{1}{16}\, t\, \sin(2t) - \dfrac{1}{8}\, t^2 \cos(2t).$

§2.5 # 14. $\Psi(t) = \dfrac{1}{6} t^3 - \dfrac{1}{4} t^2 + \dfrac{3}{4} t - \dfrac{1}{2} t\, e^{-2t}.$