

Differential Equations with Linear Algebra (M427J)

Some answers to practice problems # 5

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§2.15 # 2. $y(t) = C_1 e^{-t} + C_2 e^{3t} + C_3 e^{4t}$.

§2.15 # 4. $y(t) = C_1 \cos(t) + C_2 \sin(t) + C_3 e^t$.

§2.15 # 6. The general solution is $y(t) = C_1 e^t + C_2 e^{-t} + C_3 \cos(t) + C_4 \sin(t)$. Using the initial values, we can find the constants. The solution is

$$y(t) = \frac{1}{2}e^{-t} + \frac{1}{2}\cos(t) + \frac{1}{2}\sin(t).$$

§2.15 # 8. If $y_1(t) = e^t \cos(t)$ is a solution, we have that $1+i$ and $1-i$ are roots of the characteristic equation. This implies that $(r-1-i)(r-1+i) = r^2 - 2r + 2$ is a factor of the characteristic equation.

Dividing $r^4 - 2r^3 + r^2 + 2r - 2$ by $r^2 - 2r + 2$, we find that the characteristic equation can be factored as

$$r^4 - 2r^3 + r^2 + 2r - 2 = (r^2 - 2r + 2)(r^2 - 1) = (r^2 - 2r + 2)(r+1)(r-1).$$

The general solution is $y(t) = C_1 e^t \cos(t) + C_2 e^t \sin(t) + C_3 e^{-t} + C_4 e^t$.

§2.15 # 9. Using the formula in page 263, we find that $\Psi(t) = \int_0^t v(t-s) \tan(s) ds$, where

$$v(t) = 1 - \cos(t).$$

We can simplify the computation, if we expand $v(t-s) = 1 - \cos(t) \cos(s) - \sin(t) \sin(s)$. We find that

$$\begin{aligned} \Psi(t) &= \int_0^t \left(1 - \cos(t) \cos(s) - \sin(t) \sin(s) \right) \tan(s) ds \\ &= \int_0^t \tan(s) ds - \cos(t) \int_0^t \sin(s) ds - \sin(t) \int_0^t \sin(s) \tan(s) ds. \end{aligned}$$

After we compute the antiderivatives, we find

$$\Psi(t) = -\ln |\cos(t)| + 1 - \cos(t) + \sin(t) \cdot \ln |\sec(t) + \tan(t)|.$$

The terms $1 - \cos(t)$ are solution of the homogeneous equation, so $\Psi(t)$ can be simplified as

$$\Psi(t) = -\ln |\cos(t)| + \sin(t) \cdot \ln |\sec(t) + \tan(t)|.$$

§2.15 # 10. Using the formula in page 263, we find that

$$\Psi(t) = \int_0^t g(t-s)v(s) ds = \int_0^t v(t-s)g(s) ds,$$

where

$$v(s) = \frac{1}{4} (e^t - e^{-t} - 2 \sin t).$$

§2.15 # 12. $\Psi(t) = \frac{2t^3}{3} - 4t - 2t \sin(t)$. (*Hint:* Use the guessing method.)

§2.15 # 14. Let $L[y] = y^{(4)} - y$. We will use the guessing method with two term. We will solve

$$L[\Psi_1] = t, \quad L[\Psi_2] = \sin(t).$$

The characteristic polynomial of L has the following roots: $\pm 1, \pm i$. This implies that the functions Ψ_1 and Ψ_2 have the following forms:

$$\Psi_1(t) = A_0 + A_1 t, \quad \Psi_2(t) = t (\bar{A}_0 \cos(t) + \bar{B}_0 \sin(t)).$$

Solving, we find $\Psi_1(t) = -t$ and $\Psi_2(t) = \frac{1}{4} t \cos(t)$. We get $\Psi(t) = \Psi_1(t) + \Psi_2(t) = -t + \frac{1}{4} t \cos(t)$.