Differential Equations with Linear Algebra (M427J)

Some answers to practice problems # 7

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§3.3 # 2. Linearly independent.

§3.3 # 4. Linearly dependent.

§3.3 # 16. We define $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$. We need to find a vector of the form $\mathbf{v}_3 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ such that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent and spans the vector space $V = \mathbb{R}^3$. First, we identify all the vectors $\mathbf{v}_3$ for which $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent. Solving

$$C_1\,\mathbf{v}_1 + C_2\,\mathbf{v}_2 + C_3\,\mathbf{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

we find three equations

$$C_1 + C_2 + C_3\,a = 0, \qquad C_1 + 3C_2 + C_3\,b = 0, \qquad C_2 + C_3\,c = 0.$$

After some algebra, we find that $C_3(2a - 2b + c) = 0$. It is impossible to have $C_3 = 0$ because we would have $C_1 = C_2 = 0$. Then, if the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent, we have that $2a - 2b + c = 0$.

In conclusion, any vector of the form $\mathbf{v}_3 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ completes a basis, as long as $2a - 2b + c \neq 0$.

Extra # 1. One possible answer is $C_1 = 1$, $C_2 = -2$, $C_3 = -4$, $C_4 = 3$.

Extra # 2. $\alpha = 2$ and $\alpha = 3$ are the only two possible answers.

Extra # 4. One possible solution is $C_1 = -4$, $C_2 = 5$, $C_3 = 1$.

Extra # 5. One possible answer is $C_1 = 12$, $C_2 = -8$, $C_3 = 1$.