Differential Equations with Linear Algebra (M427J)

Some answers to practice problems # 9

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Prof. Hector E. Lomeli

§3.6 # 10. $\begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}$.

§3.6 # 12. The inverse matrix does not exists.

§3.6 # 14. The inverse matrix does not exists.

§3.7 # 4. $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

§3.7 # 6. $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

§3.7 # 8. $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

§3.7 # 10. The system has a non-trivial solution if and only if the determinant of the matrix is 0. We get $\lambda = -3, 1, 2$.

§3.7 # 12. The system has a non-trivial solution if and only if the determinant of the matrix is 0. We get $\lambda = 1$.

Extra # 1. $\begin{bmatrix} -8 & 2 & -17 \\ 9 & 1 & 11 \\ -5 & -2 & -9 \end{bmatrix}$.

Extra # 2. $\begin{bmatrix} 1 & a & a^2 - b \\ 0 & 1 & a \\ 0 & 0 & 1 \end{bmatrix}$.

Extra # 3. $\begin{bmatrix} 2 & 0 & 2a & -4a - 2b \\ 0 & -1 & 0 & 3 \\ 0 & 0 & -2 & 4 \\ 0 & 0 & 0 & 2 \end{bmatrix}$.

Extra # 4. Finding $(A - 2I_3)^2$, we get

$$(A - 2I_3)^2 = \begin{bmatrix} -1 & 1 & -1 \\ 6 & 0 & 6 \\ -2 & -1 & -2 \end{bmatrix} \cdot \begin{bmatrix} -1 & 1 & -1 \\ 6 & 0 & 6 \\ -2 & -1 & -2 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 9 \\ -18 & 0 & -18 \\ 0 & 0 & 0 \end{bmatrix}.$$

Suppose that $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ is a solution of $(A-2I_3)^2\mathbf{x}$. The we find the coordinates of $\mathbf{x}$ satisfy the equation $x_1+x_3 = 0$. We choose $x_1 = C_1$ and $x_2 = C_2$. Then $x_3 = -C_1$ and hence

$$\mathbf{x} = \begin{bmatrix} C_1 \\ C_2 \\ -C_1 \end{bmatrix} = C_1 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

We conclude that the vectors $\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ form a basis of V. The vector space V has dimension $\dim(V) = 2$.