1. Roll 4 dice. What’s the probability that you roll 4 different numbers? For example, this happens if you roll 1, 3, 4, 5 or 3, 4, 5, 2 but not 3, 3, 2, 1.

**Solution.** \[
\frac{6 \times 5 \times 4 \times 3}{6^4} = \frac{6!}{2!6^4}.
\]

2. There are 10 balls in an urn, 4 are red, 6 are blue. You select four balls uniformly at random (without replacement). What’s the probability that all 4 selected balls have the same color?

**Solution.** \[
1 + \binom{6}{4} \binom{10}{4}.
\]

3. There are 4 blue socks, 4 red socks, 4 purple socks and 4 green socks in a drawer. You grab 8 socks at random.

   (a) What is the probability of getting all of the blue socks?

   **Solution.** \[
   \binom{12}{4} \binom{16}{8}.
   \]

   (b) What is the probability of getting all of the blue socks and all of the red socks?

   **Solution.** \[
   \frac{1}{\binom{16}{8}}.
   \]

   (c) What is the probability that at least 4 of the socks you grab all have the same color?

   **Solution.** Let \( E_i \) be the event of getting all of the \( i \)-colored socks (we’ve numbered the colors 1 through 4). Then

   \[
P(E_i) = \binom{12}{4} \binom{16}{8},
   \]

   \[
P(E_i \cap E_j) = \frac{1}{\binom{16}{8}}
   \]

   (if \( i \neq j \)). So by inclusion-exclusion

   \[
P(\cup_i E_i) = 4 \binom{12}{4} \binom{16}{8} - \binom{4}{2} \frac{1}{\binom{16}{8}}.
   \]

4. Pick 4 cards at random from a standard deck. What is the probability of choosing all 4 aces conditioned on picking at least 3 aces?

   **Solution.**

   \[
   \frac{1}{\binom{3}{4} \binom{48}{1} + \binom{4}{4} \binom{48}{0}}.
   \]
5. Of the patients in a hospital, 20% of those with, and 35% of those without myocardial infarction have had strokes. If 40% of the patients have had myocardial infarction, what percent of the patients have had strokes?

**Solution.** Let $M$ denote the set of patients with myocardial infarction and $S$ the set of patients who have had a stroke. Then $P(S|M) = 0.2$, $P(S|M^c) = 0.35$, $P(M) = 0.4$. The question asks for $P(S) =$? Well

$$P(S) = P(S \cap M) + P(S \cap M^c)$$

$$= P(S|M)P(M) + P(S|M^c)P(M^c) = 0.2 \times 0.4 + 0.35 \times 0.6 = 0.08 + 0.21 = 0.29.$$

6. A collection of 65 coins contains one 2-headed coin; the remainder of the coins are fair. (A fair coin lands on heads with probability 1/2.) If a coin is selected at random from the collection, turns up heads six times in six tosses, what is the probability that it is the 2-headed coin?

**Solution.** Let $T$ denote the event of choosing the 2-headed coin and let $H$ denote the event that the coin turns up heads six times in six tosses. Then

$$P(T|H) = \frac{P(H|T)P(T)}{P(H|T)P(T) + P(H|T^c)P(T^c)} = \frac{1 \times (1/65)}{1 \times (1/65) + (1/2)^6(64/65)} = \frac{1}{1 + 1} = 1/2.$$

7. Suppose $E_1, E_2, E_3$ are jointly independent events with $P(E_1) = 0.1$, $P(E_2) = 0.2$, $P(E_3) = 0.3$. What is $P(E_1 \cup E_2 \cup E_3)$? Do not get unions confused with intersections!

**Solution.**

$$P(E_1 \cup E_2 \cup E_3) = 1 - P(E_1^c \cap E_2^c \cap E_3^c)$$

$$= 1 - P(E_1^c)P(E_2^c)P(E_3^c) = 1 - (0.9)(0.8)(0.7) = 1 - 0.504 = 0.496.$$

**Alternative solution.** Use inclusion-exclusion:

$$P(E_1 \cup E_2 \cup E_3) = \sum_{i=1}^{3} P(E_i) - \sum_{1 \leq i < j \leq 3} P(E_i \cap E_j) + P(E_1 \cap E_2 \cap E_3)$$

$$= 0.1 + 0.2 + 0.3 - (0.1)(0.2) - (0.1)(0.3) - (0.2)(0.3) + (0.1)(0.2)(0.3).$$