

Name _____

Quiz #1

- (1) A club with 10 members needs to select a President, Vice President, Secretary and Treasurer. These must be different members; for example the President cannot also be the Secretary. In how many ways can this be done?

Answer. $10 \cdot 9 \cdot 8 \cdot 7 = \frac{10!}{(10-4)!} = P_{10,4} = 5040.$

- (2) A club with 10 members needs to select 4 of its members to be on a committee.

- (a) In how many ways can this be done?

Answer. $\binom{10}{4} = C_{10,4} = \frac{10!}{4!6!} = 10 \cdot 3 \cdot 7 = 210.$

- (b) How many ways can they select the committee if two of the members (named Bob and Carol) are feuding and refuse to serve together?

Answer. $\binom{10}{4} - \binom{8}{2}$. This is because $\binom{8}{2}$ is the number of ways that Bob and Carol can both serve. Note

$$\binom{10}{4} - \binom{8}{2} = C_{10,4} - C_{8,2} = 210 - 28 = 182.$$

Alternatively, this is

$$\binom{8}{3} + \binom{8}{3} + \binom{8}{4} = 2C_{8,3} + C_{8,4}$$

because there are $\binom{8}{3}$ ways that Bob can serve without Carol, $\binom{8}{3}$ ways that Carol can serve without Bob and $\binom{8}{4}$ ways that neither serve.

- (3) A student has to sell 2 books from a collection of 6 math, 7 science and 4 economics books. How many choices are possible if

- (a) both books are to be on the same subject?

Answer. $\binom{6}{2} + \binom{7}{2} + \binom{4}{2} = 15 + 21 + 6 = 42.$

- (b) the books are to be on different subjects?

Answer. This is the number of ways to select any two books minus the number of ways to select them from the same subject. So it is

$$\binom{17}{2} - \binom{6}{2} - \binom{7}{2} - \binom{4}{2} = \binom{17}{2} - 42 = 136 - 42 = 94.$$

Alternatively, it is

$$6 \cdot 7 + 7 \cdot 4 + 4 \cdot 6 = 42 + 28 + 24 = 94$$

because there are $6 \cdot 7$ ways to choose one math and one science book, and so on.