M380C Fall 2019, Memory Jogger for Exam 1

Instructions: Put away your book and notes and try to answer the following questions from memory. If you struggle with a question, circle it. After you're done with this list, pick up the textbook and try to answer your circled questions.

1. Explain the isomorphism theorems.

2. Explain the orbit-stabilizer theorem. Give at least 4 different examples of group actions and apply the orbit-stabilizer theorem to each.

3. Explain the class equation. What does it have to do with p-groups? simplicity of A_n?

4. What are the conjugacy classes of S_n? of A_n?

5. Give some examples of generating sets of S_n.

6. Explain and prove the Jordan-Hölder Theorem.

7. Explain the semi-direct product construction and give at least 4 different examples.

8. Explain Sylow's Theorems. Prove them.

9. Define abelian, nilpotent and solvable groups. Define the ascending/descending central series and derived series. What are the main results about abelian, nilpotent and solvable groups?

10. Construct the free group $\langle a, b \rangle$. What are the main properties of free groups? Give some examples of group presentations.