TEXAS DIFFERENTIAL EQUATIONS 2020
ABSTRACTS

(1) **Andras Balogh** [UT Rio Grande Valley]. *High performance computations for a nonlinear wave equation with black hole embedded in an expanding universe*

ABSTRACT: Numerical simulations are presented for a nonlinear wave equation with singularity that models a black hole and with time dependent coefficient that models an expanding universe. The high-performance parallel computations on a CUDA platform use an explicit fourth order Runge-Kutta scheme on the temporal discretization and fourth order finite difference discretization in the 3-dimensional space. The properties of solutions with compact support are examined both inside and outside of the blackhole.

(2) **Dambaru Bhatta** [UT Rio Grande Valley]. *Hydrothermal convective flow in a porous layer in presence of a contaminant.*

ABSTRACT. Here, we investigate hydrothermal convective flow in a horizontal porous layer with rigid boundaries in presence of contaminant concentration. This flow system is governed by a set of partial differential equations consisting of the continuity equation for conservation of mass, Darcy law for conservation of momentum, heat equation for conservation of energy and the mass transport equation. Assuming a no-flow basic state system and applying weakly nonlinear approach, we derive the linear and first-order systems. We present a relationship between the critical thermal Rayleigh number and the critical contaminant Rayleigh number.

(3) **Animesh Biswas** [Iowa State U]. *Regularity theory for nonlocal space-time master equations.*

ABSTRACT. We analyze recent novel regularity theory for fractional power of parabolic operators in divergence form. These equations are fundamental in continuous time random walk models and appear as generalized Master equation. These equations are non-local in nature and were studied by Luis Caffarelli and Luis Silvestre. We developed a parabolic method of semigroups that allows us to prove a local extension problem. As a consequence we obtain interior and boundary Harnack inequalities and sharp interior and global parabolic Schauder estimates. For the latter, we also prove a characterization of the correct intermediate parabolic Hölder spaces in the spirit of Sergio Campanato. This is a part of my Ph.D dissertation jointly done with Marta de León-Contreras (Universidad of Reading, UK) and Pablo Raúl Stinga (Iowa State University).

(4) **Youn-Sha Chan** [University of Houston - Downtown]. *A crack problem modeled under strain gradient elasticity of bi-Helmholtz type.*

ABSTRACT. A crack problem is solved under a higher order of strain gradient elasticity theory of bi-Helmholtz type, and it leads to a linear partial differential equation (PDE)

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of sixth order. The sixth order PDE can be viewed as a composition of two differential operators: a second order Navier operator coming from the classical linear elasticity theory, and a fourth order bi-Helmholtz operator due to the higher order strain gradient elasticity. The bi-Helmholtz operator consists of two length scales, L1 and L2. The sixth order PDE of the crack problem is transformed to a hypersingular integral equation by the Fourier transform, and the corresponding integral equation is discretized by using the collocation method and a Chebyshev polynomial expansion. The numerical results include displacement profiles, strain, and stress fields under various combinations of L1 and L2.

(5) **Jacky Chong** [UT Austin]. *Dynamical Hartree-Fock-Bogoliubov approximation of interacting Bose gas.*

Abstract. We consider a many-body Boson system with pairwise particle interaction given by $N^{3\beta-1}v(N^{\beta}x)$ for $0 < \beta < 1$ and v a non-negative spherically-symmetric function. Our main result is the extension of the local-in-time Fock space (norm) approximation of the exact dynamics of squeezed states proved in Grillakis and Machedon, Comm. PDEs, (2017) for $0 < \beta < \frac{2}{3}$ to a global-in-time norm approximation for $0 < \beta < 1$. Our work can also be viewed as a generalization of the results of Boccato, Cenatiempo, and Schlein, Ann. Henri Poincare, (2017) to a more general set of initial data that includes coherent states along with an improved error estimate.

(6) **Erica de la Canal** [UT Austin]. *Propagation of solutions to Boltzmann systems in Banach Spaces.*

Abstract. When working with a mixture of monatomic gases, each species can be statistically described by its own distribution function, changing due to collisions with particles of the same or different species. Each of this functions satisfies a Boltzmann-like equation, and therefore we have to deal with a system of Boltzmann equations. Following recent proof of existence and uniqueness of vector value solution to this system of Boltzmann equations, by means of generation and propagation of polynomial and exponential estimates, I will present the gain of integrability and propagation of polynomially and exponentially weighted L^p norms of such a solution. This is a joint work with Irene Gamba and Milana Pavić-Čolić.

(7) **Neil Jerome A. Egarguin** [University of Houston]. *Active Manipulation of Acoustic Fields in Homogeneous Constant Depth Ocean Environments*

Abstract: In this talk, we propose an active control strategy for acoustic fields in shallow water environments. The problem is to find the necessary boundary input (either pressure or normal velocity) on a single source so that the corresponding unique solution of the exterior Helmholtz problem approximates a prescribed field in various bounded regions in the near field and prescribed far field pattern values in multiple directions. We proved that such boundary input and the corresponding unique Helmholtz field can be written in terms of a density function on the surface of a fictitious source compactly embedded in the physical source. Further, that such mapping between the density and the Helmholtz field has a dense range. We also provide numerical simulations supporting our theoretical analyses. This work can lead to interesting applications such as cloaking and shielding

against SONAR, underwater imaging and enhanced underwater communications.

(8) **Baofeng Feng** [UT Rio Grande Valley]. *Breather and rogue wave solutions to the short pulse equations.*

ABSTRACT. In this talk, we will first give a review on how to construct general breather solutions to the soliton equations via the KP hierarchy reduction method. Then we will construct multi-breather solutions to the complex short pulse equations. Finally, we will show how to get general rogue wave solution to the complex short pulse equation.

(9) **Anahit Galstyan** [UT Rio Grande Valley]. *Blow up of solutions of the semilinear wave equation in the Einstein-de Sitter spacetime.*

ABSTRACT. We present some results on the semilinear massless waves propagating in the Einstein-de Sitter spacetime. We examine the solutions of the semilinear wave equation, and, in particular, of the φ^p model of quantum field theory in the curved spacetime. More precisely, for $1 < p < 4$ we prove that solution of the massless self-interacting scalar field equation in the Einstein-de Sitter universe has finite lifespan.

(10) **Eleftherios Gkioulekas** [UT Rio Grande Valley]. *The effect of the asymmetric Ekman term on the phenomenology of the two-layer quasigeostrophic model*

ABSTRACT: In the two-layer quasi-geostrophic model, the friction between the flow at the lower layer and the surface boundary layer, placed beneath the lower layer, is modeled by the Ekman term, which is a linear dissipation term with respect to the horizontal velocity at the lower layer. The Ekman term appears in the governing equations asymmetrically; it is placed at the lower layer, but does not appear at the upper layer. A variation, proposed by Phillips and Salmon, uses extrapolation to place the Ekman term between the lower layer and the surface boundary layer, or at the surface boundary layer. We present theoretical results that show that in either the standard or the extrapolated configurations, the Ekman term dissipates energy at large scales, but does not dissipate potential enstrophy. It also creates an approximately symmetric stable distribution of potential enstrophy between the two layers. The behavior of the Ekman term changes fundamentally at small scales. Under the standard formulation, the Ekman term will unconditionally dissipate energy and also dissipate, under very minor conditions, potential enstrophy at small scales. However, under the extrapolated formulation, there exist small "negative regions", which are defined over a two-dimensional phase space, capturing the distribution of energy per wavenumber between baroclinic energy and barotropic energy, and the distribution of potential enstrophy per wavenumber between the upper layer and the lower layer, where the Ekman term may inject energy or potential enstrophy.

(11) **Kun Gou** [Texas A&M U - San Antonio]. *Computational Modeling of Thickening of Arterial Intimal tissue*

ABSTRACT: Morphoelasticity is applied to study the growth of the inner layer of arterial blood vessels. Such growth modeling is employed to understand how certain pathologies (e.g., atherosclerosis) of the arterial wall develop. The arterial wall is modeled as a three-layered hyperelastic irregular tube. Fibers are also incorporated in the wall tissue to generate

anisotropy. Due to the large length scale of the arterial wall compared with the cross-section of the tube, the deformation of the tube is taken to be axially uniform. We consider a 2-D cross-section model of the arterial wall. Nonlinear finite element computational techniques under penalty method are designed for this modeling. Various computational results are demonstrated to show how the arterial wall grows under different growth and material parameters to provide assistance for deeper understanding of corresponding arterial growth diseases.

(12) **Guangze Gu** [Central South U, P. R. China]. *Existence of positive ground state solutions for some class of non-autonomous fractional Kirchhoff equation*

ABSTRACT. In this talk, we study the existence of positive ground states for the non-autonomous fractional Kirchhoff equation with subcritical growth or three times growth. The proof is based on the concentration compactness principle and the method of Nehari manifold.

(13) **Changfeng Gui** [UT San Antonio]. *A variant of Aubin-Onofri-Inequalities on the Sphere*

ABSTRACT: I will present a variant of Aubin-Onofri inequality on the unit sphere which is in spirit similar to the Moser-Trudinger inequality on the sphere. This is a joint work with Alice Chang from Princeton University.

(14) **Qi Han** [Texas A&M U - San Antonio]. *On the differential independence of ζ and Γ*

ABSTRACT: It is a profound result of Otto Hölder in 1887 saying that the Euler gamma-function Γ cannot satisfy any nontrivial algebraic differential equation having polynomial coefficients in $\mathbb{C}$. David Hilbert, in his lecture addressed to the International Congress of Mathematicians at Paris in 1900 for his famous 23 problems, stated in Problem 18 that the Riemann zeta-function ζ can not satisfy any nontrivial algebraic differential equation having polynomial coefficients in $\mathbb{C}$, either; this problem was generalized around the 1930's by Mordukhai-Boltovskoi and Ostrowski independently, and further extended by Voronin. In 2007, Lawrence Markus, using essentially the main idea of Hölder, showed that $\zeta(\sin(2\pi z))$ and $\Gamma(z)$ are algebraically differential independent. He further conjectured that $\zeta(z)$ and $\Gamma(z)$ are algebraically differential independent as well; that is, when

$$P(\zeta, \zeta', \ldots, \zeta^{(m)}; \Gamma, \Gamma', \ldots, \Gamma^{(n)})(z) \equiv 0$$

for a polynomial P of $m+n+2$ independent variables, then necessarily $P = 0$. In this talk, we study this question and show a positive partial answer to it for some differential equations generated through a special family of functions $F(u_0, u_1, \ldots, u_m; v_0, v_1, \ldots, v_n)$, which are continuous in $(u_0, u_1, \ldots, u_m)$ but have polynomial coefficients in $(v_0, v_1, \ldots, v_n)$. This is a joint work with Dr. Jingbo Liu.

(15) **Yeyao Hu** [UT San Antonio]. *Interior and Boundary bubblings of a mean field equation with Neumann boundary condition.*

ABSTRACT: We study a mean field equation on a smooth bounded region with zero Neumann boundary condition. We constructed an assembly of boundary bubbles and interior bubbles. The locations of bubbles are determined by the global geometry of the region, namely the Neumann Green's function, its regular part and a remnant function after one removes the fundamental solution and a reflection of the fundamental solution. The scales

of bubbles not only depend on the global geometry but also depend on the local geometry, i.e., curvatures of boundary bubbling points. It is striking to emphasize that the blow-up rates increase dramatically if the assembly admits boundary bubbles. The Leray-Schauder degree associated with the equation is also computed.

(16) **Farzana Hussain** [Huston Tillotson U]. *Estimation of Maximum Water Levels Along East Coast of India due to Interaction of Storm Surge and Tide – Application to Cyclone Phailin (2013).*

ABSTRACT. Appropriate early warning systems for severe tropical cyclones and response networks are needed to ensure the safety of life and property along the coastal region. In this study a transformed coordinate boundary-fitted shallow water model is developed to predict the associated surge levels. Using appropriate transformations of independent coordinates, the curvilinear physical domain is transformed to a square one and also each island boundary transforms to a rectangle within this square domain. The vertically integrated shallow water equations are transformed to the new space domain and then the regular explicit finite difference scheme is used to solve the shallow water equations, where the problem domain is divided into 100 x 131 grid points. The model is applied to compute the water levels due to astronomical tide and surges associated with Cyclone Phailin (2013) along the East coast of India. The computed results are found in good agreement with the observations.

(17) **Ilija Jegdic** [Texas Southern U]. *Numerical simulations for conservation laws using the large time step method.*

ABSTRACT. We present a novel large time step finite volume method for hyperbolic conservation laws. In the scalar case it is proved that if the approximate solutions converge boundedly, then they converge to the entropy solution. We illustrate the method on several examples including the Keyfitz-Kranzer system where the initial data results in a nonclassical solution and contains a singular shock. This work is joint with Richard Sanders (University of Houston).

(18) **Qinfeng Li** [UT San Antonio]. *Interior approximation of sets of finite perimeter and applications to traces and extensions of divergence measure fields and BV functions*

ABSTRACT: In this talk, I will present new smooth interior approximation results for sets of finite perimeter. As an application, we can generalize previous results on extensions and traces of BV functions and divergence measure fields. This talk combines several joint works with Guiqiang Chen, Changfeng Gui, Yeyao Hu and Monica Torres.

(19) **Francesco Maggi** [UT Austin]. *Soap films and capillarity theory*

ABSTRACT. We introduce a model for soap films that uses three-dimensional regions rather than two-dimensional surfaces, and discuss some of its basic properties and open questions. This is joint work with Antonello Scardicchio (ICTP Trieste), Salvatore Stuvard and Darren King (UT Austin).

(20) **Christina Martinez** [Del Mar College]. *A study of one-dimensional flame propagation model via a system of nonlinear differential equations*

ABSTRACT. One-dimensional boundary value problem of the premixed flame propagation in a condensed medium leads to a coupled system of highly nonlinear ordinary differential equations. The reduced system depicts the transfer of two variables: temperature, θ and species, Y through a medium. In this presentation, some explicit solutions obtained via matched asymptotic expansion methods will be shown. Our focus is centered on one of the key parameters associated with the system, namely, the Lewis number ϵ_{Le}, which is the ratio of thermal to mass diffusion. It is found that the Lewis number has a greater influence on the behavior of the temperature and species. For arbitrary ϵ_{Le}, a general relation between θ and Y , in a quadrature form, is derived. This general relation is employed to validate some of the existing solutions for non-unity Lewis numbers. When $\epsilon_{Le} = 1$, the system admits an exact (approximate) solution. Our results predict the expected behavior of the temperature and species associated with one dimensional flame propagation model very well.

(21) **Taoufik Meklachi** [Penn State Harrisburg]. *Full Expansion of Resonances of 3D high contrast linear and non-linear media with an asymptotic approximation*

ABSTRACT: Scattering resonances arise in characterizing the rates of oscillation and decay of waves in scattering experiments. They are of central importance in nearly all areas of physics. We addressed the problem of computing resonances of 3D high contrast linear and non-linear media with an asymptotic approximation. In previous work we derived first order approximation. In this work a full expansion is derived.

(22) **Mallikarjunaiah Muddamallappa** [Texas A&M U - Corpus Christi]. *On quasi-static crack propagation in nonlinear elastic solids*

ABSTRACT. In this talk we describe a formulation of quasi-static crack propagation in nonlinear elastic solids. The class of constitutive relationships considered to model the response of elastic solids gives rise to nonlinear relationship between classical linearized elastic strain and Cauchy stress, which is not possible within the customary linearized elasticity. Then, I will describe an energy minimization under constraint to describe evolution of crack. An iterative algorithm will be described for the solution of the model. Numerical results will be presented to show the efficacy of the model.

(23) **Jonathan Stanfil** [Baylor U]. *Determining values of spectral $\boldsymbol{\zeta}$-functions for Sturm–Liouville operators*

ABSTRACT. Under appropriate hypotheses on regular as well as singular Sturm–Liouville operators associated with differential expressions of the type $\tau = r^{-1}[-(d/dx)p(d/dx) + q]$, the associated spectral ζ-function can be found through complex contour integration techniques to equal the residue of explicit functions, F (essentially, Fredholm determinants), involving a canonical system of fundamental solutions of $\tau y = zy$. The asymptotic behavior of F allows for a deformation of contours to find explicit expressions for the spectral ζ-function for $n \in \mathbb{N}$. We end by providing various concrete examples. If time permits we will illustrate how this can be used to understand interesting sums related to different special function, in particular, the sums of the inverse even powers of Bessel

function zeros. This is based on joint work with Guglielmo Fucci, Fritz Gesztesy, and Klaus Kirsten.

(24) **Pablo Raúl Stinga** [Iowa State U]. *On nonlocal Monge–Ampère equations.*

ABSTRACT: We report on recent progress on some nonlocal Monge–Ampère equations. Our results include the interior Harnack inequality for the fractional linearized Monge–Ampère equation (with Diego Maldonado from Kansas State University) and the regularity for the obstacle problem for the Caffarelli–Charro fractional Monge–Ampère equation (with Yash Jhaveri from Institute for Advanced Study).

(25) **Logan Stokols** [UT Austin]. *SQG on bounded domains.*

ABSTRACT. The surface quasigeostrophic (SQG) equation is of interest both for its physical relevance to meteorology and climate science, and for its interesting mathematical properties. While well-posedness for this equation is well known on $\mathbb{R}^2$ and on the torus, more general domains Ω are only recently studied. We will discuss a new model (proposed by Constantin and Ignatova) for SQG on bounded subsets of $\mathbb{R}^2$, including existence and regularity up to the boundary.

(26) **Salvatore Stuvard** [UT Austin]. *An existence theorem for mean curvature flow with fixed boundary conditions, with applications to Plateau's problem*

ABSTRACT. A one-parameter family $\{\Gamma_t\}_{t\geq 0}$ of n-dimensional surfaces in $\mathbb{R}^{n+1}$ moves by mean curvature if the velocity of the flow at any given point and time equals the mean curvature of the surface passing through that point at that time. The mean curvature flow (MCF) is arguably the most fundamental extrinsic geometric flow, due to its interpretation as the gradient flow of the area functional. In this talk, I will address the following problem: does there exist a MCF $\{\Gamma_t\}_{t\geq 0}$ of hypersurfaces in a strictly convex, bounded domain $U_0 \subset \mathbb{R}^{n+1}$ starting from any given (possibly singular) hypersurface Γ_0 and having the property that the boundary of Γ_t on ∂U_0 remains fixed along the flow? Physically, this may be interpreted as a soap film hanging from a wire which evolves "by steepest descent" towards its equilibrium configuration. I will show that, under some mild assumptions on the initial configuration Γ_0, the answer is affirmative, in the sense that a weak, measure-theoretic solution of the MCF starting from Γ_0 and keeping its boundary on ∂U_0 fixed exists for all times $t \geq 0$. This flow is shown to subsequentially converge, as $t \uparrow \infty$, to a (generalized) minimal surface with the prescribed boundary. This is joint work with Yoshihiro Tonegawa (Tokyo Institute of Technology).

(27) **Ray Treinen** [Texas State U]. *The existence of an energy minimizing configuration for multiple solid objects floating in a bath of three liquids.*

ABSTRACT. We consider the physical configuration of a container which holds a finite number of movable solid objects and three immiscible fluids. Each fluid volume is prescribed, and the container is completely filled. The configuration is modeled using sets of finite perimeter, and the energy of the configuration is given using the theory of functions of bounded variation. We show that there exists an admissible configuration that attains the minimum energy.

(28) **Vesselin Vatchev** [UT Rio Grande Valley]. *Decomposition of Two-Soliton Solutions for KdV and the Good Boussinesq Equations*

Abstract: We consider a decomposition of two-soliton solutions for KdV and the 'good' Boussinesq equations obtained by the Hirota method and by using the Wronskian technique. The explicit forms of the components allow deriving properties that can be used to better understand the dynamics of two-soliton solutions. An interpretation in the context of eigenvalue problems and transport equations is considered.

(29) **Mary Vaughan** [Iowa State U]. *A priori estimates for fractional powers of nondivergence form elliptic operators.*

Abstract. In this talk, we will discuss some ongoing work regarding regularity estimates for fractional powers of nondivergence form elliptic operators. These equations appear in the theories of elasticity and semipermeable membranes, probability, and in regards to the fractional Monge-Ampère equation of Caffarelli–Charro. We will consider some of the tools used to prove local boundedness and Harnack inequalities. This is joint work with my advisor Pablo Raúl Stinga.

(30) **Weihong Xie** [Central South U, P. R. China]. *Multiple positive solutions for the critical Kirchhoff type problems*

Abstract. Using the Nehari manifold and fibering maps, it has been proved that the critical Kirchhoff type problems admit at least two positive solutions. Furthermore, by the mountain pass theorem and Ekeland's variational principle, it is shown that the problems possess at least three positive solutions.

(31) **Karen Yagdjian** [UT Rio Grande Valley]. *Small data wave maps in cyclic spacetime*

Abstract. We study the initial value problem for the wave maps defined on the cyclic spacetime with the target Riemannian manifold that is responsive to the parametric resonance phenomena. In particular, for arbitrary small and smooth initial data we construct blowing up solutions of the wave map if the metric of the base manifold is periodic in time. This work was done in collaboration with Anahit Galstyan and Nathalie M. Luna-Rivera at University of Texas Rio Grande Valley.