

AN INTRODUCTION TO BRANCHED TRANSPORT

A. MARCHESE

MAIN REFERENCES

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FURTHER RECOMMENDED READINGS ON BRANCHED TRANSPORT AND RELATED TOPICS

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