ANNOTATED PUBLICATION LIST

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[30] Anthony Conway, Irving Dai, Maggie Miller, *$\mathbb{Z}$-disks in $\mathbb{C}P^2$*, arXiv: 2403.10080 [math.GT], Mar. 2024.

We classify locally flat disks in $(\mathbb{CP}^2)^\circ := \mathbb{CP}^2 \setminus \mathring{B}^4$ that have complements with infinite cyclic fundamental group (i.e. "$\mathbb{Z}$-disks" in $(\mathbb{CP}^2)^\circ$). Such a disk is always topologically isotopic rel. boundary to a disk obtained by performing one positive crossing change to the boundary knot K (via blowing up with $\mathbb{CP}^2$) to obtain a knot with trivial Alexander polynomial, which we then cap off with a $\mathbb{Z}$-disk. The relevant crossing change determines a loop in $S^3 \setminus \nu(K)$ and we prove that the homology class of its lift to the infinite cyclic cover of $S^3 \setminus \nu(K)$ leads to a complete invariant of the obtained disk. We prove that this determines a bijection between the set of rel. boundary topological isotopy classes of $\mathbb{Z}$-disks with boundary K and a quotient of the set of unitary units of the ring $\mathbb{Z}[t^{\pm}]/(\Delta_K)$. We deduce that a knot K with quadratic Alexander polynomial bounds 0,1,2,4, or infinitely many $\mathbb{Z}$-disks in $(\mathbb{CP}^2)^\circ$. This leads to the first examples of knots bounding infinitely many topologically distinct disks whose exteriors have the same fundamental group and equivariant intersection form. Finally we give several examples where these disks are realized smoothly.

[29] Mark Hughes, Seungwon Kim, Maggie Miller, *Branched covers of twist-roll spun knots*, arXiv: 2402.11706 [math.GT], Feb. 2024.

We prove that the double branched cover of a twist-roll spun knot in S^4 is smoothly preserved when four twists are added, and that the double branched cover of a twist-roll spun knot connected sum with a trivial projective plane is preserved after two twists are added. As a consequence, we conclude that the members of a family of homotopy $\mathbb{CP}^2$s constructed by Miyazawa are each diffeomorphic to $\mathbb{CP}^2$. As a consequence, a family of involutions constructed my Miyazawa are all defined on $\mathbb{CP}^2$ are topologically but not smoothly equivalent to complex conjugation.

[28] Andrew J. Lobb, Maggie Miller, Arunima Ray, *Morphisms in Low Dimensions*, Oberwolfach Reports **20**(1):215–259, 2023.

This is a collection of extended abstracts written by the many speakers (including lightning talk speakers) at MFO Workshop #2304, "Morphisms in Low Dimensions," which I co-organized with Andrew Lobb and Arunima Ray. This workshop brought together experts on interrelated topics in low-dimensional topology. Our goal was to improve community understanding of recent developments in the field and to promote new advances in the study of global properties of 4-manifolds.

[27] Mark Hughes, Seungwon Kim, Maggie Miller. *Non-isotopic splitting spheres for a split link in S^4*, arXiv:2307.12140 [math.GT], July 2023. Submitted.

In this paper, we extend the methods of [20] to prove that many 2-component split links in S^4 have nonunique splitting spheres. Specifically, we show that if L is a 2-component unlink and one component of L has genus at least 4, then there exist two smooth splitting spheres S_1, S_2 for L that are not even isotopic through locally flat embeddings in $S^4 \setminus \nu(L)$. This contradicts intutiton from the classical dimension, since any two smooth splitting spheres for a 2-component split link in S^3 are smoothly isotopic in the link complement. In order to prove this result, we actually study knotted 3-spheres in S^5. We construct a nontrivial 3-sphere P in S^5 so that a standard Morse function on S^5 restricts to a Morse function on P with exactly one local minimum and exactly one local maximum. We then appropriately define the Bing double of P and obtain a nonsplit link of two 3-spheres $P_1 \sqcup P_2$ in S^5 so that a standard Morse function on S^5 again restricts to each P_i as a Morse function with exactly one local minimum and one local maximum. We then obtain a 2-component unlink in S^4 by considering the intersection of $P_1 \sqcup P_2$ with an equatorial S^4 in S^5. The simplicity of the restricted height function on $P_1 \sqcup P_2$ allows us to construct two splitting spheres for the surface link in S^4, and the fact that $P_1 \sqcup P_2$ is not split allows us to distinguish these spheres.

[26] Paolo Aceto, Nickolas A. Castro, Maggie Miller, JungHwan Park, András Stipsicz, *Slice obstructions from genus bounds in definite 4-manifolds*, arXiv:2303.10587 [math.GT], Mar. 2022. Submitted.

In Summer 2022, Dai–Kang–Mallick–Park–Stoffregen proved that the (2,1)-cable of the figure eight knot is not smoothly slice, using an obstruction coming from involutive Floer homology. (They showed that if this knot were slice, then a certain involution on a homology 3-sphere would extend over a restricted rational ball, but obstructed such an extension by considering the involutive Floer complex on that homology 3-sphere.) This was particularly interesting because the (2,1)-cable of the figure eight knot has been known not to be ribbon since the 1990s by work of Casson–Gordon and Miyazaki. In this paper, we give an alternate proof of the fact that the (2,1)-cable of the figure eight knot is not smoothly slice by using a slice surface for this knot to construct a surface in $2\mathbb{CP}^2$ and applying minimum-genus bounds due to Bryan in the 1990s. We discuss this obstruction to sliceness for other knots and pose some specific problems about the minimal genus function in $2\mathbb{CP}^2$, whose positive answers would imply that various non-ribbon knots are also not slice.

[25] Maggie Miller, *Explicitly describing fibered 3-manifolds through families of singularly fibered surfaces*, arXiv:2306.13081 [math.GT], Jan. 2023. To appear in Proceedings of Symposia in Pure Mathematics “Frontiers in Geometry and Topology.”

In this paper, I give an explicit depiction of a fibration of the complement of the closure of a homogeneous braid, understanding how each fiber intersects every cross-section of S^3. This is a discussion of Stalling's classical observation that closures of homogeneous braids are fibered. While this is abstractly very easy to prove (as the fibers are Murasugi sums of torus link fibers), and with modern techniques it is usually easy to decide whether any given knot is fibered, I personally find it difficult to conceptualize the whole fibration of a knot or link complement rather than a single fiber. I wrote this analysis while attempting to extend the techniques of [3] to other 4-dimensional problems, as I felt it was necessary to understand a 3-dimensional analogue more concretely in preparation.

[24] Michael Klug and Maggie Miller, *Concordance of spheres in 4-manifolds with an immersed dual sphere*, arXiv:2211.07177 [math.GT], Nov. 2022. Submitted.

Based on work of Stong, we classify concordance of a pair of homotopic 2-spheres S_0, S_1 in a 4-manifold when S_0 has an immersed dual. When this dual has even Euler number, the obstruction to concordance is just the Freedman–Quinn invariant, as proved by Freedman–Quinn (see also [5, 12]). When the dual has odd Euler number, there is a secondary obstruction due to Stong. We give much exposition on the construction of this secondary invariant (which we call stong, in contrast to Stong's choice of “Kervaire–Milnor”) with many figures and carefully discuss the examples of [12]. We intend to write another paper on the relationship between stong and obstructing $\pi_3(W^5)$ classes from being realized by an embedded 3-sphere, for W a 5-manifold.

[23] Jason Joseph, Jeffrey Meier, Maggie Miller, Alexander Zupan, *Bridge trisections and Seifert solids*, arXiv:2210.09669 [math.GT], Oct. 2022. To appear in Algebraic & Geometric Topology.

This paper is a continuation of the work in [21]. We give a version of Seifert's algorithm for surfaces in S^4 using triplane diagrams. We show how to explicitly produce a Heegaard diagram of a Seifert surface for any 2-knot S given a triplane diagram of S. We also give some classification results, showing that if a surface Σ in S^4 admits a b-bridge trisection with at least $b-1$ disks in one sector, then Σ is unknotted. This answers a question of the second and fourth co-authors from 2017.

[22] Kyle Hayden, Seungwon Kim, Maggie Miller, JungHwan Park, Isaac Sundberg, *Seifert surfaces in the 4-ball*, arXiv:2205.15283 [math.GT], May 2022. Submitted.

We construct pairs of same-genus Seifert surfaces in S^3 with common boundary that do not become isotopic in B^4 when their interiors are pushed into B^4. Almost all examples of same-genus but non-isotopic Seifert surfaces for a knot in the literature do become isotopic when pushed into B^4 (we explain this more concretely in the introduction of the paper). The question of whether this is a general phenomenon was suggested by Livingston in 1982; this paper answers that question. (Note that this question of Livingston was a low-dimensional motivation for [20], which motivated my continued interest in the problem solved in this paper.)

We give examples where the surfaces are not topologically isotopic in B^4, with obstruction the intersection form of 2-fold covers of B^4 branched along the surfaces. We also give examples that are topologically but not smoothly isotopic, with smooth obstruction the associated cobordism maps on Khovanov homology (and topological isotopy due to work of Conway and Powell).

[21] Jason Joseph, Jeffrey Meier, Maggie Miller, Alexander Zupan, *Bridge trisections and classical knotted surface theory*, Pacific Journal of Mathematics **319**(2):343–369, 2022.

We adapt well-studied topics in knotted surface theory to the relatively new setting of bridge trisections. In particular, we give a trisection-theoretic proof of the Whitney–Massey theorem (the classification of possible Euler numbers of surfaces embedded in S^4) and show how to compute the fundamental group and related invariants of a surface from a triplane diagram.

[20] Mark C. Hughes, Seungwon Kim, and Maggie Miller, *Knotted handlebodies in the 4-sphere and 5-ball*, arXiv:2111.13255 [math.GT], Nov. 2021. To appear in Journal of the European Mathematical Society.

We show that for every $g \geq 2$, there exist genus-g 3-dimensional solids H_1 and H_2 smoothly embedded in S^4 so that the following are true.

(1) H_1 and H_2 have the same boundary.

(2) H_1 and H_2 are homeomorphic rel boundary.

(3) H_1 and H_2 are not isotopic rel boundary even through a locally flat (rather than smooth) isotopy, even when their interiors are pushed into B^5.

This is a strong resolution to the case $g \geq 2$ of a conjecture of Budney–Gabai, who posited that for all $g \geq 0$ there exist genus-g solids H_1 and H_2 smoothly embedded in S^4 that have the same boundary and are homeomorphic rel boundary but not smoothly isotopic rel boundary in S^4. Budney–Gabai proved this was true for $g = 0$; we prove this is true in an extremely strong sense (since H_1 and H_2 do not become isotopic when pushed into B^5) for $g \geq 2$. This last point is particularly surprising, since the following lower-dimensional question remained open at the time this paper was written: Does there exist a knot K in S^3 bounding two genus-g Seifert surfaces F_1 and F_2 so that F_1 and F_2 do not become (smoothly/locally flatly) isotopic rel boundary when their interiors are pushed into B^4? (Note: this question was answered in my later joint work with Hayden, Kim, Park and Sundberg [22].)

[19] Mark C. Hughes, Seungwon Kim, and Maggie Miller, *Band diagrams of immersed surfaces in 4-manifolds*, arXiv:2108.12794 [math.GT], Aug. 2021. Submitted.

We show that if Σ is a surface smoothly immersed in a 4-manifold X with isolated tranverse self-intersections, then Σ can be described by a banded singular link inside a Kirby diagram for X. We show that if Σ' is (isotopic/regularly homotopic/homotopic) to Σ, then the diagrams for Σ and Σ' are related by a certain explicit finite list of moves. We accomplish this by restricting an ambient Morse function on X to the complement of Σ and investigating flow lines of this Morse function between boundary and interior critical points. As an application, we show that a surface immersed in a trisected manifold can be put into trisection bridge position uniquely up to standard moves. We also use these singular banded diagrams to reprove some classical theorems about surfaces in 4-manifolds diagrammatically. This paper extends the work in [4].

[18] Kyle Hayden, Alexandra Kjuchukova, Siddhi Krishna, Maggie Miller, Mark Powell, and Nathan Sunukjian, *Brunnian exotic surface links in the 4-ball*, arXiv:2106.13776 [math.GT], June 2021. To appear in Michigan Mathematical Journal.

For any natural number $n \geq 2$, we construct n-component surfaces Σ_1, Σ_2 smoothly, properly embedded into B^4 with the property that each Σ_i is *Brunnian* (meaning that if you delete any one component of Σ_i you will get a surface that is smoothly isotopic rel. boundary to a Seifert surface for an unlink) and Σ_1, Σ_2 are an *exotic pair* (meaning that Σ_1 and Σ_2 are topologically isotopic rel. boundary but are not smoothly equivalent).

This is an example of a very subtle form of exotic behavior. If you delete one component from each of Σ_1 and Σ_2, the resulting surfaces are smoothly isotopic rel. boundary. Thus, an operation transforming Σ_1 into Σ_2 must somehow make use of every component of Σ_1, rather than being a local operation. When $n = 2$, we obstruct sliceness using an adjunction inequality or tools from knot Floer homology, in two different constructions. We then give an argument via branched coverings to reduce exoticness of $(n+1)$-component surfaces to n-component surfaces.

The surfaces Σ_1 and Σ_2 can be chosen to consist only of disks. However, if we allow Σ_1 and Σ_2 to have a positive-genus component, then we can extend the result to construct infinitely many surfaces $\{\Sigma_m\}_{m\in\mathbb{N}}$ which are pairwise exotic. This argument makes use of the tools from [13].

[17] Maggie Miller and Burak Ozbagci, *Lefschetz fibrations on nonorientable 4-manifolds*, Pacific Journal of Mathematics **312**(1):177–202, 2021.

We show that a nonorientable 4-dimensional manifold built without 3- or 4-handles admits a Lefschetz fibration over the disk. In the relative nonorientable seting, regular fibers of a Lefschetz fibration are nonorientable surfaces with boundary. As a corollary, we obtain a 4-dimensional proof of the fact that every nonorientable closed 3-manifold admits an open book decomposition, which was first proved by Berstein and Edmonds using branched coverings. This paper was motivated by the work in [16].

[16] Maggie Miller and Patrick Naylor, *Trisections of non-orientable 4-manifolds*, arXiv:2010.07433 [math.GT], Oct 2020. To appear in Michigan Mathematics Journal.

We show that several classical theorems are true in the non-orientable setting, including Waldausen's theorem (i.e. that Heegaard splittings of $\#_g S^2 \widetilde{\times} S^1$ are standard) and Laudenbach-Poenaru's theorem (i.e. that any diffeomorphism of $\partial(\natural_k B^3 \widetilde{\times} S^1)$ extends to a diffeomorphism of $\natural_k B^3 \widetilde{\times} S^1$). This implies that Kirby diagrams and trisection diagrams of closed non-orientable 4-manifolds exist and are unique up to standard moves. Without a non-orientable Laudenbach-Poenaru theorem, one must explicitly describe attaching regions of 4-dimensional 3-handles in any surgery diagram, so this greatly improves our ability to describe non-orientable manifolds. We also describe how to use trisections to take covers and to describe embedded surfaces (as in the orientable case). In the case of Cappell-Shaneson homotopy 4-spheres (which are double covers of exotic $\mathbb{RP}^4$s), this yields potentially nonstandard trisections of standard S^4 along with trisections of potential exotic S^4s that may be easier to analyze via trisection diagrams than with previously existing methods.

[15] Maggie Miller and Alexander Zupan, *Equivalent characterizations of handle-ribbon knots*, arXiv:2005.11243 [math.GT], May 2020. To appear in Communications in Analysis and Geometry.

For the purpose of this paper, a knot is *slice* if it bounds a disk smoothly embedded into a homotopy 4-ball. A derivative of a knot K is a link L sitting on a Seifert surface for K in a prescribed way; a knot K has a derivative if and only if K is algebraically slice. We study handle-ribbon (i.e. strongly homotopy-ribbon) knots, which bound slice disks whose complements admit handle decompositions with no 3-handles.

The stable Kauffman conjecture posits that a knot is slice if and only if it admits a slice derivative. We prove that a knot is handle-ribbon if and only if it admits an R-link derivative; i.e. an n-component derivative L with the property that zero-framed surgery on L yields $\#^n(S^1 \times S^2)$. We also show that K bounds a handle-ribbon disk D in a homotopy 4-ball B if and only if the 3-manifold obtained by zero-surgery on K admits a singular fibration that extends over handlebodies in $B \setminus \nu(D)$, generalizing a classical theorem of Casson and Gordon about fibered homotopy-ribbon knots to all handle-ribbon knots. The techniques in Section 5 of this paper are very similar to those in [3]. Other sections in this paper rely on the theory of Morse 2-functions.

[14] Paolo Aceto, Jeffrey Meier, Allison N. Miller, Maggie Miller, JungHwan Park, and András I. Stipsicz, *Branched covers bounding rational homology balls*, Algebraic & Geometric Topology. **21**(7):3569–3599, 2021.

If a knot is slice, then all of its prime-powered branched covers bound rational homology balls. This is a very strong restriction and has been used to obstruct sliceness for large families of knots (e.g. Lisca used this property to understand sliceness of 2-bridge knots, and then Greene and Jabuka used this property to understand sliceness of odd 3-stranded pretzels).

In this paper, we use twisted Alexander polynomials to prove that four knots (the 3-strand Turk's head knots with 14, 22, 34, and 46 crossings, respectively) are linearly independent in the knot concordance group, yet all of their prime-powered branched covers bound rational homology balls. None of these knots are of the form (or obviously concordant to a knot of the form) $K\#K^r$, which is the previously known way of constructing such knots (suggested by Chuck Livingston). Conjecturally, there should be an infinitely generated subgroup of such concordance classes in the knot concordance group. Due to the difficulty of computing twisted Alexander polynomials, a strategy for finding an infinite basis of such knots is unclear.

[13] András Juhász, Maggie Miller, and Ian Zemke, *Transverse invariants and exotic surfaces in the 4-ball*, Geometry & Topology **25**(6):2963–3012, 2021.

We produce infinitely many genus-one surfaces properly embedded in B^4 that are pairwise topologically isotopic rel. boundary but not smoothly equivalent. The topological isotopy is constructive, although uses Freedman's theorem that knots with trivial Alexander polynomial are topologically slice. The obstruction to smooth equivalence comes from the knot Floer concordance maps (or more precisely, maps on perturbed stured Floer homology after suitably decorating the surfaces), which are sensitive to smooth topology.

[12] Michael R. Klug and Maggie Miller, *Concordance of surfaces and the Freedman-Quinn invariant*, Journal of Topology **14**(2):560–586, 2021.

We extend a concordance version of the 4-dimensional light bulb theorem to the case where one lightbulb has an immersed dual. That is, we prove that if R, R' are homotopic π_1-trivial (i.e. $\pi_1(R), \pi_1(R')$ include trivially into $\pi_1(X)$) surfaces smoothly embedded in a 4-manifold X and G is a framed immersed 2-sphere intersecting R geometrically once, then R and R' are concordant modulo a condition on 2-torsion in $\pi_1(X)$. This 2-torsion condition comes from the Freedman-Quinn invariant of a pair of based-homotopic 2-spheres in a 4-manifold, which has recently been studied by Schneiderman and Teichner. (When R and R' are 2-spheres, this comes down to assuming the invariant for the pair is zero; when R and R' are positive genus then we have to say this in terms of a given homotopy from R to R'. See [24] for more details. In particular, it is sufficient to assume that $\pi_1(X)$ has no 2-torsion. This generalizes [5].) For R and R' 2-spheres, this follows also from work of Freedman–Quinn.

We give explicit counterexamples when G is not framed, using Stong's Kervaire-Milnor invariant (defined for some pairs of based-homotopic spheres). This is the first construction of a pair of spheres with nontrivial Stong (Kervaire-Milnor) invariant.

[11] Nickolas A. Castro, Gabriel Islambouli, Maggie Miller, and Maggy Tomova, *The relative $\mathcal{L}$-invariant of a compact 4-manifold*, Pacific Journal of Mathematics **315**(2):305–346, 2021.

We introduce the *relative $\mathcal{L}$-invariant* $r\mathcal{L}(X)$ of a smooth, orientable, compact 4-manifold X with boundary. This trisection-theoretic invariant is motivated by the definition of the $\mathcal{L}$-invariant for smooth, orientable, closed 4-manifolds by Kirby and Thompson. We show that if X is a rational homology ball, then $r\mathcal{L}(X) = 0$ if and only if $X \cong B^4$.

We also prove that any two relative trisections of a given 4-manifold X are related by interior stabilization, relative stabilization, and the relative double twist, which we introduce in this paper as a trisection version of one of Piergallini and Zuddas's moves on open book decompositions. Previously, it was only known (by Gay and Kirby) that relative trisections inducing equivalent open books on X are related by interior stabilizations. This paper completes the uniqueness classification of relative trisections.

[10] Irving Dai and Maggie Miller, *The 0-concordance monoid is infinitely generated*, Proceedings of the American Mathematical Society **151**(8):3601–3609, 2023

We study the relation of 0-concordance on 2-knots in S^4. This is a restricted type of concordance introduced by Melvin to study the Gluck twist; if K and J are 0-concordant 2-knots, then their Gluck twists are diffeomorphic. Equivalence classes of 2-knots under 0-concordance form the elements of a monoid $\mathcal{M}_0$, with operation connected-sum. Sunukjian recently showed that $\mathcal{M}_0$ is infinite by showing that if K_0 and K_1 are 0-concordant and bound punctured rational homology spheres $\mathring{Y}_i$, then certain d-invariants of Y_0 and Y_1 agree. We extend his argument to show that in fact there is a spin rational homology cobordism from Y_0 to Y_1. By appealing to results about spin rational homology cobordism (or integer homology cobordism, if adaptable), we show that $\mathcal{M}_0$ is infinitely generated.

[9] Maggie Miller, *The effect of link Dehn surgery on the Thurston norm*, International Mathematics Research Notices **2023**(22):19069–19114. 2023.

Let $L = L_1 \sqcup L_2$ be a 2-component link in S^3 with nonzero linking number. Assume that $X := S^3 \setminus \mathring{\nu}(L)$ has nondegenerate Thurston norm. Let F be a norm-minimizing surface in X, and let $\widehat{F}$ be the closed surface in $S^3_{\partial F}(L)$ obtained from F by capping off each boundary component with a disk. Baker and Taylor showed (in a more general setting) that there is a finite set of slopes S on $\partial\nu(L_1)$ so that if $\partial F \cap L_1$ is not a slope in S, then $\widehat{F} \setminus \mathring{\nu}(L_2)$ is norm-minimizing in $S^3_{\partial F \cap L_1}(L_1)$. In contrast, I study the effect of simultaneously Dehn-filling both boundary components of X.

In this paper, I show that there is a finite set $E \subset H_2(X, \partial X; \mathbb{Z})$ so that if $[F]$ is primitive and outside E, then $\widehat{F}$ is norm-minimizing in $S^3_{\partial F}(L)$. More generally, I show that when L is a multi-component link in a rational homology sphere Y (with nonzero linking numbers and nondegenerate Thurston norm), a norm-minimizing surface F remains norm-minimizing after Dehn surgery according to ∂F as long as $[F]$ lies outside of an $(n-2)$-dimensional set of rays in $H_2(Y \setminus \nu(L), \partial; \mathbb{R})$.

The main argument of this paper involves constructing an explicit superset of E. I am very interested in producing a smaller (or sharp) superset.

[8] András Juhász, Maggie Miller, and Ian Zemke, *Knot cobordisms, torsion, and Floer homology*, Journal of Topology **13**(4):1701–1724, 2020.

Let K and J be knots in S^3. We show that v-torsion Ord_v in the $\mathcal{F}_2[v]$-modules $HFK^-(K), HFK^-(J)$ constrain the geometry of cobordisms between J and K. In particular, if there is a ribbon concordance from J to K with n local minima, then $\mathrm{Ord}_v(K) \leq \max\{n, \mathrm{Ord}_v(J)\}$. When J is the unknot, then K is ribbon and this yields $\mathrm{Ord}_v(K) \leq \mathrm{Fus}(K)$, the fusion number of K. Applying simple algebraic properties of Ord_v, we conclude that for any knot L, $\mathrm{Ord}_v(L) \leq \mathrm{br}(L)$, the bridge-index of L.

This is the first result in the literature that relates knot Floer homology to the bridge index of a knot.

[7] Peter Lambert-Cole and Maggie Miller, *Trisections of 5-manifolds*, 2019 MATRIX Annals, MATRIX Book Ser., Springer, pp. 117–134, 2021.

Rubinstein and Tillmann constructed multisections of PL n-manifolds, which are decompositions of a PL n-manifold into $\lceil (n+1)/2 \rceil$ elementary pieces with well-understood intersections. When $n = 3$, a multisection is a Heegaard splitting; when $n = 4$, a multisection is a trisection. In this paper, we show how to construct a multisection of a smooth 5-manifold from a handle decomposition. We show that any smooth cobordism between trisected 4-manifolds can be multisected to agree with the trisections on its boundary.

Although not explicitly written in this paper, the argument also shows how to 4-sect any closed 5-manifold X. That is, how to write $X = Y_1 \cup Y_2 \cup Y_3 \cup Y_4$ with the Y_i's intersecting only at their boundary and for $i \neq j \neq k$ we have that Y_i is a 5D 1-handlebody, $Y_i \cap Y_j$ is a 4D 1-handlebody, $Y_i \cap Y_j \cap Y_k$ is a 3D 1-handlebody, and $Y_1 \cap Y_2 \cap Y_3 \cap Y_4$ is a closed surface. To see why this is true, just fix a handle decomposition of X and let Y_1 be the union of all 0- and 1-handles. Then view $W := X \setminus \mathring{Y}_1$ as a cobordism from ∂Y_1 to $\emptyset$ and use the results of this paper to obtain a trisection (Y_2, Y_3, Y_4) of W with the desired properties.

[6] Maggie Miller and Ian Zemke, *Knot Floer homology and strongly homotopy-ribbon concordances*, Mathematical Research Letters **28**(3):849–861, 2021.

Ian Zemke previously showed that knot Floer homology can obstruct a ribbon concordance by proving that if K is ribbon-concordant to J, then there is an injection $\widehat{HFK}(K) \hookrightarrow \widehat{HFK}(J)$ (as a summand). We extend this argument to a generalization of ribbon concordance due to Cochran. (See listing [15].) Specifically, say K is strongly homotopy-ribbon concordant to J if there is an annulus in a homotopy $S^3 \times I$ between K and J whose complement can be built from $S^3 \setminus \mathring{\nu}(K)$ by attaching 4-dimensional 1- and 2-handles. Using handle calculus, we show that this condition is sufficient to apply the previous techniques of Zemke and obtain an injection $\widehat{HFK}(K) \hookrightarrow \widehat{HFK}(J)$ (again as a summand). In September 2020, Gujral and Levine proved a version of this theorem for Khovanov homology.

[5] Maggie Miller, *A Concordance Analogue of the 4D Light Bulb Theorem*, International Mathematics Research Notices **2021**(4):2565–2587, 2021.

I consider a concordance analogue of David Gabai's 4-dimensional light bulb theorem. Gabai's light bulb theorem says that when R and R' are smoothly embedded, homotopic 2-spheres in a 4-manifold X and admit a common transverse sphere (i.e. a 2-sphere in X with trivial normal bundle which meets R and R' each exactly once, transversely) then R and R' are ambiently isotopic, modulo a condition on 2-torsion in $\pi_1(X)$. In this paper, I weaken the hypothesis of the light bulb theorem at the cost of weakening the conclusion from ambient isotopy to concordance. More precisely, I show that if R and R' are homotopic and R has a transverse sphere, then R and R' are concordant, modulo the same condition on 2-torsion in $\pi_1(X)$. The proof relies on part of the proof of the 4-dimensional light bulb theorem. Gabai shows that if R' is in a certain normal form, then R' is isotopic to R. I show that in the weakened hypotheses, R' is concordant to a 2-sphere in this normal form.

[4] Mark C. Hughes, Seungwon Kim, and Maggie Miller, *Isotopies of surfaces in 4-manifolds via banded unlink diagrams*, Geometry & Topology **24**(3):1519–1569, 2020.

We show that banded unlink diagrams of a surface Σ in a 4-manifold X – combinatorial descriptions of the restriction of a Morse function on X to Σ – are unique up to a finite set of explicit moves. As an application, we show that bridge trisections of surfaces are unique up to a perturbation move, confirming a conjecture of Meier and Zupan. This goes through the original proof of Meier and Zupan, who showed that bridge trisections exist by constructing them from banded unlink diagrams. We also produce several interesting isotopies of spheres in the generating homology class of $\mathbb{CP}^2$. We show that for many large families of 2-knots K known to have Gluck twist diffeomorphic to S^4, the 2-sphere $K\#\mathbb{CP}^1 \subset \mathbb{CP}^2$ is isotopic to $\mathbb{CP}^1$. (It holds by work of Melvin that $(\mathbb{CP}^2, K)$ is diffeomorphic to $(\mathbb{CP}^2, \mathbb{CP}^1)$.)

[3] Maggie Miller, *Extending fibrations on knot complements to ribbon disk complements*, Geometry & Topology **25**(3):1479–1550, 2021.

I show that if K is a fibered ribbon knot in S^3 bounding ribbon disk $D \subset B^4$, then if D satisfies a transversality condition with respect to the fibration on $S^3 \setminus \mathring{\nu}(K)$ then the fibration extends to a fibration of $B^4 \setminus \mathring{\nu}(D)$ by handlebodies. This is related to a question of Casson and Gordon, who showed that K bounds some disk E in a homotopy 4-ball V so that $V \setminus \mathring{\nu}(E)$ is fibered by handlebodies, and so asked whether V can be taken to be B^4. When D has two local minima, the transversality condition is automatically satsified and we conclude that $B^4 \setminus \mathring{\nu}(D)$ is fibered. More generally, I construct a library of twenty simple circular Morse functions with no interior critical points on small 4-manifolds and investigate when these movies can be concatenated to find a circular Morse function with no critical points on a larger 4-manifold (via Cerf theory).

This was my PhD thesis.

[2] Seungwon Kim and Maggie Miller, *Trisecting surface complements and Price twist surgery*, Algebraic & Geometric Topology **20**(1):343–373, 2020.

We show how to produce a relative trisection description of the complement of any surface smoothly embedded in a smooth 4-manifold. The main tool is a new move on the boundary of a relative trisection. Deleting a bridge-trisected surface from a trisected 4-manifold will in general not yield a relatively trisected manifold, but after performing this boundary move finitely many times, this complement will be relatively trisected. As an application, we show how to produce a trisection description of the result of the Price twist, a certain surgery operation on embedded $\mathbb{RP}^2$s which can yield exotic 4-manifolds.

[1] Maggie Miller, *Concordances from the standard surface in $S^2 \times S^2$*, Journal of Knot Theory and its Ramifications **29**(9):1950-57, 2019.

I use Gabai's 4-dimensional light bulb theorem in $S^2 \times S^2$ to construct concordances between homologous surfaces in $S^2 \times S^2$. The existence of these concordances follows from work of Sunukjian. Here we make use of the lightbulb theorem to see the concordances explicitly. This is a short, diagrammatic paper.