Here are some Putnam- or Putnam-like questions that can at least be *stated* to a freshman Calculus student. (I would not expect a typical freshman to have much idea of how to solve them, though!)

1. Find the value of b for which

$$\lim_{x \to 0} \left(\frac{1/(3x+b) - 2}{x} \right)$$

exists; then, find the value of this limit.

2. Evaluate this integral:

$$\int_{x=0}^{x=1} \frac{x^4(1-x)^4}{1+x^2}\, dx$$

3. Show that this improper integral converges:

$$\int_0^\infty \sin(x)\, \sin(x^2)\, dx$$

4. Evaluate

$$\sum_{n=2}^{\infty} \log\left(\frac{n^3-1}{n^3+1} \right)$$

5. Evaluate

$$\int_0^\infty \frac{\arctan(\pi x) - \arctan(x)}{x}\, dx$$

6. Suppose that f is differentiable and that $f'(x)$ is strictly increasing on $[0, \infty)$. Suppose further that $f(0) = 0$. Prove that $g(x) = f(x)/x$ is strictly increasing on $(0, \infty)$

7. Let f be a three times differentiable function (defined on $\mathbf{R}$ and real-valued) such that f has at least five distinct real zeros. Prove that $f + 6f' + 12f'' + 8f'''$ has at least two distinct real zeros.
(This is an actual Putnam question from last year!)

8. Find the minimum value of

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

among positive numbers x, y, z subject to $x + y + z = 1$.

9. Prove that $n! < \left(\dfrac{n+1}{2} \right)^n$ for $n = 2, 3, 4, \ldots$

10. Show that

$$(a^2b + b^2c + c^2a)(ab^2 + bc^2 + ca^2) \geq 9a^2b^2c^2$$

for all positive real numbers a, b, c