More Exotic Options

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2. Compound Options
3. Gap Options

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Definition; Some types

- The payoff of a Barrier option is path dependent
- More precisely, the payoff depends on whether over the option life the underlying price reaches the barrier; note that this is a simplistic view of things - the stock prices are observed at discrete times and the wording above implies continuous observation of stock prices
- Knock-out options go **out of** existence if the asset price reaches the barrier; the variants are

↓ down-and-out: has to fall to reach the barrier

↑ up-and-out: has to rise to reach the barrier

- Knock-in options come **into** existence if the asset price reaches the barrier; the variants are

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Rebate options

- Rebate options make a fixed payment if the asset price reaches the barrier; we have
  - ↓ down rebates: has to fall to reach the barrier
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- More complex barrier options require the asset price to not only cross a barrier, but spend a certain amount of time across the barrier in order to knock in or knock out

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Up-and-Out Call

- Let us spend some time studying a particular barrier option - the analysis of the other options would be analogous
- Assume that the underlying risky asset S is a geometric Brownian motion

$$dS(t) = rS(t)\,dt + \sigma S(t)\,d\tilde{W}(t),$$

where $\tilde{W}$ is a standard Brownian motion under the risk-neutral measure $\tilde{\mathbb{P}}$
- Consider a European call, expiring at time T with strike price K and up-and-out barrier B (of course, $K < B$)
- As we have seen before, the closed form for the solution of the above SDE for the asset price S is

$$S(t) = S(0)e^{\sigma\tilde{W}(t)+(r-\frac{1}{2}\sigma^2)t} = S(0)e^{\sigma\hat{W}(t)}$$

where we introduced the stochastic process

$$\hat{W}(t) = \frac{1}{\sigma}(r - \frac{1}{2}\sigma^2)t + \tilde{W}(t)$$

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Up-and-Out Call: On the maximum of S

- Define

$$\hat{M}_T = \max_{0 \leq t \leq T} \hat{W}(t)$$

- Then, we have that

$$\max_{0 \leq t \leq T} S(t) = e^{\sigma \hat{M}(T)}$$

- The option kicks out if and only if

$$S(0)e^{\sigma \hat{M}(T)} > B$$

- If the above happens - the option is rendered worthless
- If

$$S(0)e^{\sigma \hat{M}(T)} \leq B$$

then the payoff is

$$(S(T) - K)^+ = (S(0)e^{\sigma \hat{W}(T)} - K)^+$$

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Up-and-Out Call: The payoff

- Altogether, when we combine the above two cases, the payoff of the option is

$$\begin{aligned} V(T) &= (S(0)e^{\sigma \hat{W}(t)} - K)^{+}\mathbb{I}_{\{S(0)e^{\sigma \hat{M}(T)} \leq B\}} \\ &= (S(0)e^{\sigma \hat{W}(t)} - K)\mathbb{I}_{\{S(0)e^{\sigma \hat{W}(t)} \geq K,\, S(0)e^{\sigma \hat{M}(T)} \leq B\}} \\ &= (S(0)e^{\sigma \hat{W}(t)} - K)\mathbb{I}_{\{\hat{W}(T) \geq k,\, \hat{M}(T) \leq b\}} \end{aligned}$$

  where

$$k = \frac{1}{\sigma}\ln\left(\frac{K}{S(0)}\right);$$

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The Valuation

- In general, barrier options are "cheaper" than the otherwise identical "ordinary" options
- Under some minor assumptions, it is possible to find the price of the barrier options in the Black-Scholes setting
- The formula for the price is quite long - but it contains only N as a special function
- In fact, the up-and-out option's price satisfies the Black-Scholes-Merton partial differential equation (as the price for the ordinary European call) - it is the boundary conditions that are different (they have to account for the barrier) and they complicate matters somewhat

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More Exotic Options

The Set-up

- A compound option is an option to buy an option
- Let us draw a timeline
- Consider a call on a call option, i.e., an option to buy a call option with maturity T and strike price K at some exercise time $T_1 < T$, for some strike price K_1
- This call on a call should be exercise at time T_1 only if the strike price K_1 is lower than the price of the underlying call option at time T_1
- So, the payoff of this option at time T_1 is

$$(C(T_1) - K_1)^+ = (C(S(T_1), K, T - T_1) - K_1)^+$$

where $C(T_1) = C(S(T_1), K, T - T_1)$ is the current price of the underlying call option

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The Pricing

- Assume the Black-Scholes-Merton setting and denote the price of the underlying call option at time T_1 by $C(T_1)$ - we know the formula for this value
- Then the time 0 price of the call on a call option can be represented as

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&\tilde{\mathbb{E}}_0[e^{-rT_1}(C(T_1)-K_1)^+] \\
&\quad = \tilde{\mathbb{E}}_0[e^{-rT_1}(\tilde{\mathbb{E}}_{T_1}[e^{-r(T-T_1)}(S(T)-K)^+]-K_1)^+] \\
&\quad = \tilde{\mathbb{E}}_0[e^{-rT}(S(T)-K)^+ - e^{-rT_1}K_1]\mathbb{I}_{\{C(T_1)\geq K_1\}}
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where $\tilde{\mathbb{E}}$ denotes the expectation with respect to the risk-neutral probability $\tilde{\mathbb{P}}$

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Parity

- Let *CallOnCall* denote the price of the compound call on an underlying call option with maturity T_1
- Let *PutOnCall* denote the price of the compound put on an underlying call option (the exact analogue of the above call-on-call)
- Let *Call* denote the price of the underlying call option
- Then the parity for compound options reads as

$$CallOnCall - PutOnCall = Call - x^{-rT_1}$$

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A Gap Call

- A gap call option pays SK_1 when $S > K_2$
- There is a gap in the payoff diagram - hence the name of the option
- The Black-Scholes price of a gap call option is

$$C(S, K_1, K_2, \sigma, r, T, \delta) = Se^{-\delta T} N(d_1) - Ke^{-rT} N(d_2)$$

where

$$d_1 = \frac{1}{\sigma\sqrt{T}} \left[\ln\left(\frac{S}{K_2}\right) + (r - \delta + \frac{1}{2}\sigma^2) T \right]$$

and $d_2 = d_1 - \sigma\sqrt{T}$

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A Gap Call

- A gap call option pays $S K_1$ when $S > K_2$
- There is a gap in the payoff diagram - hence the name of the option
- The Black-Scholes price of a gap call option is

$$C(S, K_1, K_2, \sigma, r, T, \delta) = Se^{-\delta T} N(d_1) - Ke^{-rT} N(d_2)$$

where

$$d_1 = \frac{1}{\sigma\sqrt{T}} \left[\ln\left(\frac{S}{K_2}\right) + (r - \delta + \frac{1}{2}\sigma^2) T \right]$$

and $d_2 = d_1 - \sigma\sqrt{T}$