The Growth of Money

1. Interest
2. Accumulation and amount functions
3. Simple Interest/Linear Accumulation Functions
4. Discount functions/The time value of money
5. Simple discount

What is interest?

- $\$K\ldots$ the principal, i.e., the amount of money that the **borrower borrows/lender lends** at time $t = 0$
- $\$S\ldots$ the amount of money that changes hands at a later time - say, T
- The interest is defined as

$$\$S - \$K \geq 0$$

Reading assignment: Section 1.2 in the textbook (on the rationale behind the existence of interest)

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The amount function

- Let us temporarily fix the principal \$$K$
- $A_K(t)$ ... the amount function for principal \$$K$, i.e., the balance at time $t \geq 0$ (time is always measured in some agreed upon units; think "years" for now)
- In words: \$$K$ invested at time $t = 0$ **"grows"** to $A_K(t)$ at time $t \geq 0$
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The accumulation function

- $a(t)$ ... the accumulation function, i.e., the amount function if the principal $\$K$ is one dollar
- *Formally:* If the principal is one dollar, we write

$$a(t) = A_1(t)$$

- Note:

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The relationship between the amount and the accumulation functions

- We expect to have that

$$A_K(t) = Ka(t)$$

- This is *common*, but is **NOT** always the case (the investment scheme may include a *tiered* growth structure).
- However, since the above equality holds in most cases, we will assume that it is true unless it is *explicitly* noted otherwise.

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The increase/decrease of the amount and the accumulation functions

- It is natural to assume that both a and A_K increase in the time variable.
- Such increase may be, for example:
  - continuous and linear;
  - discrete (end of the year, e.g.);
  - continuous and exponential
- However, there are investment schemes in which it is possible to **lose** money over time (e.g., if one invests in a fund that trades in the market or in a restaurant that takes time to pay off)
- *Assignment:* Examples 1.3.2-4 in the textbook.

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The effective interest rate

Let $t_2 > t_1 \geq 0$

- $A_K(t_2) - A_K(t_1) \ldots$ the amount of interest earned between time t_1 and time t_2
- $i_{[t_1,t_2]} \ldots$ the effective interest rate for the interval $[t_1, t_2]$, i.e.,

$$i_{[t_1,t_2]} = \frac{a(t_2) - a(t_1)}{a(t_1)}$$

- **IF** $A_K(t) = Ka(t)$, then we also have

$$i_{[t_1,t_2]} = \frac{A_K(t_2) - A_K(t_1)}{A_K(t_1)}$$

- The interval $[n-1, n]$ is called the n^{th} time period (for n a positive integer)
- *Notation:*

$$i_n = i_{[n-1,n]} = \frac{a(n) - a(n-1)}{a(n-1)}$$

- Hence,

$$a(n) = a(n-1)(1 + i_n)$$

and $i_1 = a(1) - 1$

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Linear $a(t)$

- In this case, a is assumed linear and, thus, must be of the form

$$a(t) = 1 + st$$

  for a certain constant s

- s ... the simple interest rate
- Note: $s = i_1$
- $A_K(t) = K(1 + st)$ ... the amount function for \$$K$ invested **by simple interest** at rate s
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On i_n

- In the simple interest case:

$$i_n = \frac{s}{1 + s(n-1)}$$

- So, i_n is **decreasing** in n (see Example 1.4.2 in the textbook for an illustration of this fact)
- Moreover,

$$i_n \to 0, \text{ as } n \to \infty$$

- So, simple interest is not convenient for long duration loans.

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Methods for measuring the time/length of the loan in years

- Exact simple interest aka "actual/actual"

  The loan term $\mathcal{D}$ expressed in days and divided by 365

- Ordinary simple interest aka "30/360"

  The loan term $\mathcal{D}$ expressed in days assuming that each month has 30 days and then divided by 360

- The Banker's rule aka "actual/360"

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The Growth of Money

The discount function

- $v(t)$... the discount function, i.e.,

$$v(t) = \frac{1}{a(t)}$$

- In words, $v(t)$ is the amount of money that one should **invest** at time 0 in order to have \$1 at time t
- For example, in the simple interest case, we have that

$$v(t) = \frac{1}{1+st}$$

- *Question:* What if one wishes to invest a certain amount not at time 0 but at a later time $t_1 > 0$ - with the goal of earning \$$S$ at a still later time t_2?
- Let us draw the time line ....
- One needs to invest (at time t_1)

$$\$Sv(t_2)a(t_1) = \$S\frac{a(t_1)}{a(t_2)} = \$S\frac{v(t_2)}{v(t_1)}$$

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Present Value

- $PV_{a(t)}(\$L \text{ at } t_0)$ ... present value with respect to $a(t)$ of $\$L$ to be received at time t_0, i.e.,

$$PV_{a(t)}(\$L \text{ at } t_0) = \$Lv(t_0)$$

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The Growth of Money

Discount rate

- D ... the discount per unit time period, per dollar that the borrower and the lender agree upon at time 0, i.e.,

  If an investor (lender) lends \$1 for one basic period at a discount rate D - this means that in order to obtain \$1 at time 0, the borrower must **pay immediately** $\$D$ to the lender.

- Note that the "net-effect" for the borrower is that they get to use

$$\$(1 - D)$$

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- The initial fee is proportional to the amount of money borrowed, i.e., if one wants to borrow $\$K$, one needs to pay $\$DK$ to the lender
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or, equivalently

$$v(t) = 1 - tD$$

- Note that the discount function $v(t)$ is linear in this case
- **Caveat:** This situation is **not** the same as the one when the accumulation function is linear.

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- More vocabulary:

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