More on The Growth of Money

1. Interest in Advance/The Effective Discount Rate

2. Compound Interest (the Usual Case!)

3. Compound Discount

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3. Compound Discount

What is interest?

- Recall the definition of the discount D per unit time period, per dollar that the borrower and the lender agree upon at time 0:

  If an investor (lender) lends \$1 for one basic period at a discount rate D - this means that in order to obtain \$1 at time 0, the borrower must **pay immediately** \$$D$ to the lender.

- We say that \$$KD$ is the **amount of discount** for the loan
- $d_{[t_1,t_2]}$... the effective discount rate for the interval $[t_1, t_2]$, i.e.,

$$d_{[t_1,t_2]} = \frac{a(t_2) - a(t_1)}{a(t_2)}$$

- Under the assumption that $A_K(t) = Ka(t)$ for every $t \geq 0$, we also have that

$$d_{[t_1,t_2]} = \frac{A_K(t_2) - A_K(t_1)}{A_K(t_2)}$$

- In particular, for the effective discount rate in the n^{th} time period we write

$$d_n = d_{[n-1,n]} = \frac{a(n) - a(n-1)}{a(n)}$$

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Examples

I. Find the net-amount of money which you would receive at time 0 were you to borrow $1000 at a simple discount rate of 9% for the period of 3 years

⇒

$$1000[1 - 0.09 \cdot 3] = 730$$

II. Let the rate of simple discount be 10%. Find d_5.

⇒ By definition

$$d_5 = \frac{a(5) - a(4)}{a(5)} = \frac{\frac{1}{1-5\cdot 0.1} - \frac{1}{1-4\cdot 0.1}}{\frac{1}{1-5\cdot 0.1}} = \frac{2 - \frac{5}{3}}{2} = \frac{1}{6}$$

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Equivalence of the rate of interest and the rate of discount

- A rate of interest and a rate of discount are said to be equivalent for an interval $[t_1, t_2]$ if they produce the same accumulated value at time t_2 for a dollar invested at time t_1.
- Simple algebra yields that the equivalence of $i_{[t_1,t_2]}$ and $d_{[t_1,t_2]}$ holds if and only if

$$1 = (1 + i_{[t_1,t_2]})(1 - d_{[t_1,t_2]})$$

i.e., if

$$i_{[t_1,t_2]} = \frac{d_{[t_1,t_2]}}{1 - d_{[t_1,t_2]}}$$

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- What do these conditions read as for the n^{th} time period rates?

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An Example

- Let d be the simple discount rate. Assume that the simple interest rate i is equivalent to the simple discount rate d over t periods. Calculate i in terms of d and t.

⇒ Note that it does not matter which interval of length t we consider. The simplest approach is to compare the accumulation functions that the two schemes produce:
By the simple interest

$$a(t) = 1 + i \cdot t$$

By the simple discount

$$\tilde{a}(t) = \frac{1}{1 - d \cdot t}$$

Equating the two, we get

$$1 + i \cdot t = \frac{1}{1 - d \cdot t}$$

i.e.,

$$i = \frac{1 - (1 - d \cdot t)}{t(1 - d \cdot t)} = \frac{d}{1 - d \cdot t}$$

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Assignment

Examples 1.6.1 and 1.6.5
Problems 1.6.1-4

More on The Growth of Money

Compound Interest

- Define

$$i = i_1 = a(1) - 1$$

- Assume that an accumulation function $a(t)$ has the associated periodic interest rates all equal, i.e., assume that

$$i_n = i \quad \text{for every positive integer } n$$

Then the accumulation function must be equal to

$$a(n) = (1+i)^n \quad \text{for every positive integer } n$$

Moreover,

$$a(t) = (1+i)^t \quad \text{for every } t \geq 0$$

- We call $a(t)$ defined above compound interest rate accumulation function at interest rate i
- The word **"compound"** means that the interest earned is automatically reinvested to earn additional interest.

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An Example

- Find the accumulated value of $2000 dollars that are invested for 4 years at the rate $i = 0.08$ using compound interest

⇒ It is not assumed otherwise, so

$$A_{2000}(4) = 2000a(4)$$

From the expression for the amount function $a(t)$ in the case of compound interest:

$$a(4) = (1 + 0.08)^4$$

Hence, $A_{2000}(4) = 2720.98$

- Note that we had the same values, but the *simple interest* scheme in our very first example. There we obtained that the accumulated value after 4 years was equal to $2640.

  The "excess" $2720.98 - 2640 = 80.98$ comes from compounding, i.e., reinvesting the obtained interest.

- *Reading Assignment:* Read the paragraph on **monetary** and **fiscal** policies on page 24 in the textbook.

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An Example: Varying interest/Unknown interest

- Assume that an investment earns 4% the first year, 5% the second year and 21.89% the third year.
- A different investment with the same principal is made at an **unknown** annual compound interest rate.
- The amounts on both accounts are the same after three years. What is the unknown annual compound interest rate?

⇒ Without loss of generality, let us set that the principal is equal to a single dollar.

The first investment scheme results in the following amount of money after three years:

$$(1 + 0.04)(1 + 0.05)(1 + 0.2189) = 1.3310$$

On the other hand, let us denote the unknown annual compound interest rate by i. We have that

$$1.3310 = (1 + i)^3$$

So, $i \approx 0.9896 = 9.896\%$

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- Assume that an investment earns 4% the first year, 5% the second year and 21.89% the third year.
- A different investment with the same principal is made at an **unknown** annual compound interest rate.
- The amounts on both accounts are the same after three years. What is the unknown annual compound interest rate?

⇒ Without loss of generality, let us set that the principal is equal to a single dollar.
The first investment scheme results in the following amount of money after three years:

$$(1 + 0.04)(1 + 0.05)(1 + 0.2189) = 1.3310$$

On the other hand, let us denote the unknown annual compound interest rate by i. We have that

$$1.3310 = (1 + i)^3$$

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Assignment

- Examples 1.5.4-8
  Problems 1.5.1-10

The discount factor v

- Recall that in the present case, the accumulation function reads as

$$a(t) = (1+i)^t \text{ for every } t \geq 0$$

- We introduce

$$v := \frac{1}{1+i}$$

and call it the discount factor

- Then the **discount function** takes the form

$$v(t) = \frac{1}{a(t)} = \frac{1}{(1+i)^t} = v^t$$

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More on The Growth of Money

1. Interest in Advance/The Effective Discount Rate

2. Compound Interest (the Usual Case!)

3. Compound Discount

The Definition

- In analogy with the case of compound interest, here we assume that the effective discount rate d_n is constant for every unit time period, i.e., we assume that there is a constant d such that

$$d = d_n \qquad \text{for every } n \geq 1$$

- Then the equivalent interest rate has the form

$$i_n = i := \frac{d}{1-d} \qquad \text{for every } n \geq 1$$

- So, in this case, the interest rate is itself constant!
- *Terminology:* If "year" is our basic time unit, then we say that d is the annual effective discount rate

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- So, in the present case, the equivalent effective discount and insurance rates satisfy the equality

$$d = \frac{i}{1+i}$$

- Using the notation for the discount factor $v = \frac{1}{1+i}$, we can rewrite the above as

$$d = iv$$

- Once you know which is which in the above notation, the last equality is the easiest one to remember that connects these three values!
- Note that also

$$d + v = 1$$

- Your calculator should be capable of recovering the other two of the above quantities if the third is given (see the recipes in the textbook)

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The compound discount accumulation function

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$$a(t) = (1 - d)^{-t}$$

  and call it compound discount accumulation function at discount rate d

- *Assignment:* Example 1.9.13 in the textbook

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