Fund Performance

An Example: “Buy-and-hold”

- Assume that Roger has a risky investment fund in which $1000 investment drops to $500 at the end of the six months, but then recovers and is again worth $1000 at the end of the year
- The yield rate pertaining to this fund is evidently equal to zero

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An Example: A profitable strategy

- Assume that Roger has the same risky investment fund in which \$1000 investment drops to \$500 at the end of the six months, but then recovers and is again worth \$1000 at the end of the year
- However, at the end of the six months, Roger decides to double the outstanding balance in the fund, i.e., contribute another \$500
- Now, the amount of money actually in the account again doubles during the latter six months
- The equation of value for this set of transactions is

$$1000(1+i) + 500(1+i)^{1/2} = 2000$$

- Hence, $i = 0.4069$

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- This time, suppose that Roger decides to withdraw half of his outstanding investment at the end of the first six months.
- At the end of the year, the balance is twice what it was after that withdrawal, i.e., \$500
- This time, the equation of value is

$$1000(1+i) - 250(1+i)^{1/2} = 500$$

  and so $i = -0.2892$
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- Consider our set-up for an investment account from the previous section (unit time length, etc.)
- We introduce another **measure** of performance of our fund: the annual time-weighted yield rate
- B_t ... denotes the **balance** on the account at time t - just before any contributions that take place at time t
- In particular, $B_0 = A$ and $B_1 = B$
- Recall that C denotes the net contributions to the fund, i.e.,

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- Assume that there are finitely many (say, r) times $0 < t_1 < t_2 < \cdots < t_r \leq 1$ at which $C_{t_l} \neq 0$; additionally, set $t_{r+1} = 1$
- Let j_k denote the effective interest rate (yield rate) for the period $[t_{k-1}, t_k]$ for every $k = 1, \ldots r+1$; then, we can write

$$1 + j_k = \begin{cases} \frac{B_{t_1}}{B_0} & \text{for } k = 1 \\ \frac{B_{t_k}}{B_{t_{k-1}} + C_{t_{k-1}}} & \text{for } k = 2, 3, \ldots r+1 \end{cases}$$

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An Example

- On January 1^{st} an investment account is worth \$100,000.
- On $May 1^{st}$, its value has increased to \$112,000 and \$30,000 more have been contributed to the account.
- On November 1^{st}, the value of the account is \$125,000 and \$42,000 is withdrawn.
- On January 1^{st} the following year, the balance on the account is again \$100,000
- Find the time-weighted yield rate.

⇒ The time weighted rate of interest is

$$j_{tw} = \frac{112,000}{100,000} \cdot \frac{125,000}{112,000 + 30,000} \cdot \frac{100,000}{125,000 - 42,000} - 1$$
$$= 1.12 \cdot 0.880282 \cdot 1.204819 - 1$$

We get $j_{tw} = 0.1879$

- *Assignment:* In the problem above, try to calculate the dollar-weighted yield and compare the two results

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