Deferred Annuities Certain

General terminology

- A deferred annuity is an annuity whose first payment takes place at some predetermined time $k+1$
- ${}_{k|n}a$... the present value of a basic deferred annuity-immediate with term equal to n and the deferral period k; it can be readily expressed as

$$ {}_{k|n}a = v^k \cdot a_{\overline{n}|} = a_{\overline{k+n}|} - a_{\overline{k}|} $$

- It makes sense to ask for the value of a deferred annuity at any time before the beginning of payments and also after the term of the annuity is completed; here we mean more than one period before and more than one period after since these two cases are easily reduced to annuities immediate and annuities due
- It will be clear what we mean after some examples ...

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An Example: Accumulated value after the last payment date

- On January 1^{st} of year y, you open an investment account. If an annuity such that twelve annual payments equal to $\$2,000$ are made starting December 31^{st} of year y is going to be credited to the account, find the account balance on December 31^{st} four years after the last annuity payment. Assume that $i = 0.05$.

An Example: Accumulated value after the last payment date (cont'd)

⇒ The payments are level, so let us start by considering a basic deferred annuity-immediate.

The accumulated value of the 12−year long annuity-immediate at the time of the last payment is

$$s_{\overline{12|}\,0.05}$$

During the following four years this value will grow to

$$(1+0.05)^4 \cdot s_{\overline{12|}\,0.05}$$

Finally, recall that each level payment equals \$2,000. So, the accumulated value we seek is

$$\begin{aligned} 2000 \cdot (1+0.05)^4 \cdot s_{\overline{12|}\,0.05} &= 2000 \cdot (1+0.05)^4 \cdot \frac{(1+0.05)^{12}-1}{0.05} \\ &= 38,694.73. \end{aligned}$$

- *Assignment:* For a similar story, see Example 3.5.2

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An Example:
Present value of a deferred annuity - The value before the term of the annuity

- Today is January 1^{st} of year y. An annuity-immediate pays $1,000 at the end of every quarter. The first payment is scheduled for March 31^{st} of year $y+1$ and the last payment for December 31^{st} of year $y+5$. Assume that the rate of interest is equal to $i^{(4)} = 0.08$. Find the **present value** of the annuity.

An Example: Present value of a deferred annuity - The value before the term of the annuity (cont'd)

⇒ It is more convenient to be thinking in terms of quarter-years. The interest rate per quarter is $j = i^{(4)}/4 = 0.02$.

The value on **January** 1^{st} **of year** $y+1$ of a basic annuity-immediate corresponding to the one in the example is

$$a_{\overline{24|}\,0.02} = \frac{1-v^{24}}{i} = 18.913.93$$

So, the present value of a basic annuity-immediate is

$$\left(\frac{1}{1.02}\right)^{4} a_{\overline{24|}\,0.02} = 17.4735$$

Finally, the present value of our level annuity-immediate is

$$1000 \cdot \left(\frac{1}{1.02}\right)^{4} \cdot a_{\overline{24|}\,0.02} = 17,473.5$$

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Assignment

- Examples 3.5.3,4
- Problems 3.5.1,2

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