Outstanding Loan Balances and Nonlevel Annuities

1. Oustanding Loan Balances

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Retrospective method

Assume that a loan amount L is supposed to be repaid over n time periods by level payments at the **end** of each period; we want to be able to find the **balance of the loan** at an intermediate time during the loan term

- k ... denotes the time at which we want to find the loan balance just **after** the time$-k$ payment
- OLB_k ... is this loan-balance, i.e.,

$$OLB_k = L \cdot a(k) - Q \cdot s_{\overline{k|}}$$

  where Q stands for the level amount of each of the first k payments and a denotes the accumulation function associated with the loan
- If i is the effective interest rate per payment period in the compound interest setting, then

$$OLB_k = L(1+i)^k - Q \cdot s_{\overline{k|}\,i}$$

- This method of "looking back", i.e., of looking how much one has repaid until time k when determining the loan-balance is called the retrospective method

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An Example

- A loan is being repaid with 10 payments of \$2,000 each followed by 10 payments of \$1,000 each at the end of each half-year. Assume that the nominal rate of interest convertible semiannually equals $i^{(2)} = 10\%$.

  Find the outstanding loan balance immediately after the fifth payment is made.

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An Example (cont'd)

⇒ Recall the general formula:

$$OLB_k = L(1+i)^k - Q \cdot s_{\overline{k|}\,i}$$

and let the basic time unit be a half-year

Then, the ingredients in the above equation are
$k = 5$, $Q = 2000$, $i = 0.05$ and the loan amount L is unknown

We can get the loan amount L as the present value of the payments that are made to repay the loan, i.e.,

$$\begin{aligned} L &= 2000 \cdot a_{\overline{10|}} + 1000 \cdot {}_{10|10}a \\ &= 1000(a_{\overline{20|}} + a_{\overline{10|}}) \\ &= 1000(12.4622 + 7.7217) = 20,184 \end{aligned}$$

So,

$$\begin{aligned} OLB_k &= 20184 \cdot 1.05^5 - 2000 \cdot s_{\overline{5|}\,0.05} \\ &= 20184 \cdot 1.27628 - 2000 \cdot 5.5256 = 14,709 \end{aligned}$$

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Prospective method

- Contrary to the retrospective method, the prospective method is based on "looking into the future", i.e., evaluating the value of remaining payments
- If the compund interest with constant rate i is used, then we have that

$$OLB_k = Q \cdot a_{\overline{n-k-1|}\, i} + R \cdot (1+i)^{k-n}$$

  where Q denotes the level amount of all but the last payment and R the amount in the last payment
- Naturally, if $Q = R$, the above equation is simpler

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An Example

- A loan is being repaid with 20 annual payments of $1,000 each. At the time of the fifth payment, the borrower wishes to repay an extra $2,000 and then repay the balance using level annual payments over the next 12 years.

  Of course, due to the extra $2,000 repaid at time 5, the level payments that are supposed to be made over those 12 years need to be **revised**. Assume that the effective rate of interest is 9%. Find the amount of the revised annual payment.

⇒ Using the prospective method, we get that the balance of the loan after the first 5 years is

$$OLB_5 = 1000 \cdot a_{\overline{15}|} = 1000 \cdot 8.06070 = 8060.70$$

Thus, after the extra $2,000 are repaid, the outstanding loan balance becomes $6,060.70

Denote the revised level payment amount by X. This amount should satisfy

$$Xa_{\overline{12}|} = 6060.70 \Rightarrow X = \frac{6060.70}{7.1607} = 846.38$$

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An Example: Adjustable Rate Mortgage

- A borrower takes out a 30−year adjustable rate mortgage for \$65,000. The interest rate for the first year is 8%. If the interest rate increases to 10% for the second year, find the increase in the monthly payments.

⇒ The monthly payment Q for the first year can be found as

$$Q = \frac{65,000}{a_{\overline{360|}\,0.08/12}} = \frac{65,000}{136.2835} = 476.95$$

So, the outstanding balance after one year is

$$OLB_{12} = 476.95 a_{\overline{348|}\,0.08/12} = 476.95 \cdot 135.1450 = 64,457.42$$

* *Which method did we use above?*

Then, the **revised** monthly payment $\tilde{Q}$ (for the second year, at least) can be obtained as

$$\tilde{Q} = \frac{64,457.42}{a_{\overline{348|}\,0.10/12}} = \frac{64,457.42}{113.3174} = 568.82$$

Finally, the increase in the monthly payments is

$$\tilde{Q} - Q = 568.82 - 476.95 = 91.87$$

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Which method should we use?

- The answer depends on the context:

  If the payments before time k are level, while the last payments are irregular, then it is better to use the retrospective method

  If you do not know the total number of payments n, again, you use the retrospetive method

  If the last payments are level and you know n, use the prospective method

- *Assignment:* Examples 3.6.5, 3.6.6, 3.6.7, 3.6.9, 3.6.10

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Amount of principal paid

- The **amount of principal paid** in any single period $[k-1, k]$ is

$$OLB_{k-1} - OLB_k$$

- The **amount of interest paid** in any single period $[k-1, k]$ is

"total payment at time k"

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An Example

- A loan is to be repaid by annual installments of P at the end of each year for 10 years. You are given the following:

(1) The amount of principal paid over the first 3 years is 290.35

(2) The amount of principal paid over the last 3 years is 408.55

Find the total amount of interest paid during the loan term.

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An Example (cont'd)

⇒ Let L denote the loan amount, and let I denote the total amount of interest paid. Then,

$$I = 10P - L$$

So, we need to find i, P and L

(1) ⇒ The amount of principal paid over the first three years is

$$\begin{aligned} OLB_0 - OLB_3 &= P \cdot (v + v^2 + \cdots + v^{10}) - P \cdot (v + v^2 + \cdots + v^7) \\ &= P \cdot (v^8 + v^9 + v^{10}) \\ &= P \cdot v^7 (v + v^2 + v^3) = 290.35 \end{aligned}$$

(2) ⇒ The amount of principal paid over the first three years is

$$\begin{aligned} OLB_7 - OLB_{10} &= P \cdot (v + v^2 + v^3) - 0 \\ &= P \cdot (v + v^2 + v^3) = 408.55 \end{aligned}$$

Dividing the above two equations, we get

$$v^7 = \frac{290.35}{408.55} \Rightarrow i = 0.05$$

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$$\begin{aligned} OLB_0 - OLB_3 &= P \cdot (v + v^2 + \cdots + v^{10}) - P \cdot (v + v^2 + \cdots + v^7) \\ &= P \cdot (v^8 + v^9 + v^{10}) \\ &= P \cdot v^7 (v + v^2 + v^3) = 290.35 \end{aligned}$$

(2) ⇒ The amount of principal paid over the first three years is

$$\begin{aligned} OLB_7 - OLB_{10} &= P \cdot (v + v^2 + v^3) - 0 \\ &= P \cdot (v + v^2 + v^3) = 408.55 \end{aligned}$$

Dividing the above two equations, we get

$$v^7 = \frac{290.35}{408.55} \Rightarrow i = 0.05$$

An Example (cont'd)

⇒ Let L denote the loan amount, and let I denote the total amount of interest paid. Then,

$$I = 10P - L$$

So, we need to find i, P and L

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An Example (cont'd)

Now, we have the interest rate $i = 0.05$
and we are ready to find P and L.

Using condition (2) again, we get

$$P = \frac{408.55}{v + v^2 + v^3} = \frac{408.55}{2.7234} = 150.02$$

Hence,

$$L = P \cdot a_{\overline{10|}\, 0.05} = 1,158.44$$

Finally, the amount of interest paid is

$$10 \cdot 150.02 - 1,158.44 = 341.66$$

- *Assignment:* Calculator work from Section 3.7
  We will cover Section 3.7 through problems ...

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