Perpetuities

General terminology and basic properties

- A perpetuity is an annuity whose payment take place forever, i.e., it is an annuity whose term is infinite
- $a_{\overline{\infty}|}$ . . . the present value of a basic perpetuity-immediate; if one wishes to emphasize the effective per-period interest rate i one can use $a_{\overline{\infty}|\,i}$; moreover, it can be readily expressed as

$$a_{\overline{\infty}|\,i} = \frac{1}{i}$$

- $\ddot{a}_{\overline{\infty}|}$ . . . the present value of a basic perpetuity-due; again, to emphasize the effective per-period interest rate i we use $\ddot{a}_{\overline{\infty}|\,i}$; moreover,

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An Example

- Roger's uncle leaves an estate of \$100,000. Interest on the estate is paid to Roger for the first 10 years, to Roger's sister Sally for the second 10 years and to the Daisy Lovers' Association thereafter. All the payments are made at the end of the year.

  Find the relative shares that Roger, Sally and DLA have in the estate, assuming that the estate earns 7% annual effective rate of interest.

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An Example (cont'd)

⇒ The estate's worth is \$100, 000. So, with the annual interest rate of $i = 0.07$, the amount of every yearly payment is \$7, 000 (regerdless of who the recepient is).

The value of Roger's share is

$$7000 \cdot a_{\overline{10}|} = 7000 \cdot 7.0236 = 49165$$

The value of Sally's share is really the present value of a deferred annuity, i.e.,

$$\begin{aligned} 7000 \cdot {}_{10|10}a &= 7000 \cdot (a_{\overline{20}|} - a_{\overline{10}|}) \\ &= 1000(10.5940 - 7.0236) = 24993 \end{aligned}$$

The value of the DLA's share is actually the present value of a deferred perpetuity, i.e.,

$$7000 \cdot (a_{\overline{\infty}|} - a_{\overline{20}|}) = 7000 \left(\frac{1}{0.07} - 10.5940 \right) = 25842$$

- Note that the three shares sum up to 100,000 - which makes sense

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