

Outstanding Loan Balances and Nonlevel Annuities

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Retrospective method

Assume that a loan amount L is supposed to be repaid over n time periods by level payments at the **end** of each period; we want to be able to find the **balance of the loan** at an intermediate time during the loan term

- $k \dots$ denotes the time at which we want to find the loan balance just **after** the time- k payment
- $OLB_k \dots$ is this **loan-balance**, i.e.,

$$OLB_k = L \cdot a(k) - Q \cdot s_{\overline{k}|i}$$

where Q stands for the level amount of each of the first k payments and a denotes the accumulation function associated with the loan

- If i is the effective interest rate per payment period in the compound interest setting, then

$$OLB_k = L(1+i)^k - Q \cdot s_{\overline{k}|i}$$

- This method of “looking back”, i.e., of looking how much one has repaid until time k when determining the loan-balance is called the **retrospective method**

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An Example

- A loan is being repaid with 10 payments of \$2,000 each followed by 10 payments of \$1,000 each at the end of each half-year. Assume that the nominal rate of interest convertible semiannually equals $i^{(2)} = 10\%$.

Find the outstanding loan balance immediately after the fifth payment is made.

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An Example (cont'd)

⇒ Recall the general formula:

$$OLB_k = L(1 + i)^k - Q \cdot s_{\overline{k}|i}$$

and let the basic time unit be a half-year

Then, the ingredients in the above equation are

$k = 5$, $Q = 2000$, $i = 0.05$ and the loan amount L is unknown

We can get the loan amount L as the present value of the payments that are made to repay the loan, i.e.,

$$\begin{aligned} L &= 2000 \cdot a_{\overline{10}|} + 1000 \cdot {}_{10|}10a \\ &= 1000(a_{\overline{20}|} + a_{\overline{10}|}) \\ &= 1000(12.4622 + 7.7217) = 20,184 \end{aligned}$$

So,

$$\begin{aligned} OLB_k &= 20184 \cdot 1.05^5 - 2000 \cdot s_{\overline{5}|0.05} \\ &= 20184 \cdot 1.27628 - 2000 \cdot 5.5256 = 14,709 \end{aligned}$$

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Prospective method

- Contrary to the retrospective method, the **prospective method** is based on “looking into the future”, i.e., evaluating the value of remaining payments
- If the compound interest with constant rate i is used, then we have that

$$OLB_k = Q \cdot \overline{a}_{n-k-1}|i + R \cdot (1+i)^{n-k}$$

where Q denotes the level amount of all but the last payment and R the amount in the last payment

- Naturally, if $Q = R$, the above equation is simpler

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- A loan is being repaid with 20 annual payments of \$1,000 each. At the time of the fifth payment, the borrower wishes to repay an extra \$2,000 and then repay the balance using level annual payments over the next 12 years.

Of course, due to the extra \$2,000 repaid at time 5, the level payments that are supposed to be made over those 12 years need to be **revised**. Assume that the effective rate of interest is 9%. Find the amount of the revised annual payment.

⇒ Using the prospective method, we get that the balance of the loan after the first 5 years is

$$OLB_5 = 1000 \cdot a_{\overline{15}|} = 1000 \cdot 8.06070 = 8060.70$$

Thus, after the extra \$2,000 are repaid, the outstanding loan balance becomes \$6,060.70

Denote the revised level payment amount by X . This amount should satisfy

$$Xa_{\overline{12}|} = 6060.70 \Rightarrow X = \frac{6060.70}{7.1607} = 846.38$$

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An Example: Adjustable Rate Mortgage

- A borrower takes out a 30-year adjustable rate mortgage for \$65,000. The interest rate for the first year is 8%. If the interest rate increases to 10% for the second year, find the increase in the monthly payments.

⇒ The monthly payment Q for the first year can be found as

$$Q = \frac{65,000}{a_{\overline{360}|0.08/12}} = \frac{65,000}{136.2835} = 476.95$$

So, the outstanding balance after one year is

$$OLB_{12} = 476.95 a_{\overline{348}|0.08/12} = 476.95 \cdot 135.1450 = 64,457.42$$

* Which method did we use above?

Then, the revised monthly payment \tilde{Q} (for the second year, at least) can be obtained as

$$\tilde{Q} = \frac{64,457.42}{a_{\overline{348}|0.08/12}} = \frac{64,457.42}{113.3174} = 568.82$$

Finally, the increase in the monthly payments is

$$\tilde{Q} - Q = 568.82 - 476.95 = 91.87$$

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Which method should we use?

- The answer depends on the context:

If the payments before time k are level, while the last payments are irregular, then it is better to use the retrospective method

If you do not know the total number of payments n , again, you use the retrospective method

If the last payments are level and you know n , use the prospective method

- *Assignment:* Examples 3.6.5, 3.6.6, 3.6.7, 3.6.9, 3.6.10

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Amount of principal paid

- The **amount of principal paid** in any single period $[k - 1, k]$ is

$$OLB_{k-1} - OLB_k$$

- The **amount of interest paid** in any single period $[k - 1, k]$ is

“total payment at time k ”

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An Example

- A loan is to be repaid by annual installments of P at the end of each year for 10 years. You are given the following:
 - (1) The amount of principal paid over the first 3 years is 290.35
 - (2) The amount of principal paid over the last 3 years is 408.55Find the total amount of interest paid during the loan term.

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An Example (cont'd)

⇒ Let L denote the loan amount, and let I denote the total amount of interest paid. Then,

$$I = P \cdot a_{\overline{10}|i} - L$$

So, we need to find i , P and L

(1) ⇒ The amount of principal paid over the first three years is

$$\begin{aligned} OLB_0 - OLB_3 &= P \cdot (v + v^2 + \dots + v^{10}) - P \cdot (v + v^2 + \dots + v^7) \\ &= P \cdot (v^8 + v^9 + v^{10}) \\ &= P \cdot v^7(v + v^2 + v^3) = 290.35 \end{aligned}$$

(2) ⇒ The amount of principal paid over the first three years is

$$\begin{aligned} OLB_7 - OLB_{10} &= P \cdot (v + v^2 + v^3) - 0 \\ &= P \cdot (v + v^2 + v^3) = 408.55 \end{aligned}$$

Dividing the above two equations, we get

$$v^7 = \frac{290.35}{408.55} \Rightarrow i = 0.05$$

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$$\begin{aligned} OLB_0 - OLB_3 &= P \cdot (v + v^2 + \dots + v^{10}) - P \cdot (v + v^2 + \dots + v^7) \\ &= P \cdot (v^8 + v^9 + v^{10}) \\ &= P \cdot v^7(v + v^2 + v^3) = 290.35 \end{aligned}$$

(2) ⇒ The amount of principal paid over the first three years is

$$\begin{aligned} OLB_7 - OLB_{10} &= P \cdot (v + v^2 + v^3) - 0 \\ &= P \cdot (v + v^2 + v^3) = 408.55 \end{aligned}$$

Dividing the above two equations, we get

$$v^7 = \frac{290.35}{408.55} \Rightarrow i = 0.05$$

An Example (cont'd)

⇒ Let L denote the loan amount, and let I denote the total amount of interest paid. Then,

$$I = P \cdot a_{\overline{10}|i} - L$$

So, we need to find i , P and L

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An Example (cont'd)

Now, we have the interest rate $i = 0.05$
and we are ready to find P and L .

Using condition (2) again, we get

$$P = \frac{408.55}{v + v^2 + v^3} = \frac{408.55}{2.7234} = 150.02$$

Hence,

$$L = P \cdot a_{\overline{10}|0.05} = 1,158.44$$

Finally, the amount of interest paid is

$$10 \cdot 150.02 - 1,158.44 = 341.66$$

- *Assignment:* Calculator work from Section 3.7
We will cover Section 3.7 through problems ...

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