# Continuously paying annuities

1. Compound interest: Increasing payments

2. General Accumulation Function

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# The Set-up: Unit increase in payments

Assume that we have compound interest with the effective interest rate per interest period equal to $i$.

Consider the following continuous annuity:

- the annuity lasts for $n$ interest periods;
- the payments take place continuously, at a rate of $t$ per interest period at time $t$.
- $(\bar{I}\bar{a})_{\overline{n}|\,i}$ ...stands for the present value of the above annuity, i.e.,

$$(\bar{I}\bar{a})_{\overline{n}|\,i} = \lim_{m\to\infty}(I^{(m)}a)^{(m)}_{\overline{n}|\,i} = \frac{\bar{a}_{\overline{n}|\,i} - nv^n}{\delta}$$

- It is easier to see what happens by noting that

$$(\bar{I}\bar{a})_{\overline{n}|\,i} = \int_0^n t \cdot v^t\, dt$$

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# Unit payment stream

- Let $v(t)$ denote the general discount function
- Let us first consider the basic continuous annuity, i.e., the annuity that pays at the unit rate at all times.
- Then, the present value of such an annuity with length $n$ equals

$$\int_0^n v(t)\,dt$$

- We still denote the above present value by $\bar{a}_{\overline{n|}}$
- In the special case of compound interest, the above formula collapses to the one already familiar to us from the compound interest set-up You can verify this through simple integration ...

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# Any payment stream

- Let $f(t)$ be a continuous function which represents the rate of payments of a continuous annuity on the time interval $[0, n]$
- Then, the present value of this annuity can be obtained as

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- *Assignment:* Problems 4.6.1,2,3,5,7

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