Bond Amortization Schedule

Bonds as Loans

- The bonds can be seen as loans that the holder of the bond gives to the issuer of the bond; the coupon payments and the redemption payment are there to repay this loan
- The coupon period plays the role of the payment period we are familiar with from the context of amortized loans
- The investor's effective yield rate per coupon period j stands for the per payment period interest rate in the set-up for loan repayment
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Notation

- $B_t, t \in [0, n]$, denotes the balance of debt at time t immediately **after** any time t coupon payment, but **before** redemption payment, i.e., the book value of the bond at time t
- *Caveat:* B_t is calculated using the **investor's** yield rate j and does not take into account market forces (e.g., the prices of bonds on the secondary market)
- In particular,

$$B_0 = P \text{ and } B_n = C$$

- I_t ...the interest due at time of the t^{th} coupon, i.e.,

$$I_t = jB_{t-1}, \; t = 1, 2, \ldots n$$

- P_t ...the amount of adjustment of principal in the t^{th} coupon, i.e.,

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Finding B_t for $t = 0, 1, 2, \ldots n$

- From the basic price formula:

$$B_t = F\, r\, a_{\overline{n-t|}\, j} + C\, v_j^{n-t} \quad t = 0, 1, 2, \ldots n$$

- From the premium-discount formula:

$$B_t = C(g - j)a_{\overline{n-t|}\, j} + C \quad t = 0, 1, 2, \ldots n$$

So,

$$I_t = jB_{t-1} = C\, g - C(g - j)\, v_j^{n-t+1}$$

and

$$P_t = B_{t-1} - B_t = C(g - j)\, v_j^{n-t+1}$$

- Hence,

$$I_t + P_t = C\, g,$$

i.e., the coupons consist of the payment of interest due and the repayment of principal

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At a premium/At a discount/$C = P$

At a premium:
If a bond sells at a premium, then $g > j$ and P_t is positive for every t. So, a portion of each coupon compensates the investor for the premium initially paid for the bond and the rest goes towards the interest payment on that premium.

At a discount:
If a bond sells at a discount, then $g < j$ and P_t is negative for every t. Thus, not even the interest due on the balance can be covered by the coupon payments meaning that the remainder of the interest due is added onto the outstanding loan balance which increases as time goes by

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If the price of the bond equals the redemption value, then $g = j$ and $P_t = 0$ for every t

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Amount for amortization of premium

- If the bond is sold at a premium, we usually refer to P_t as the amount for amortization of premium
- In this case the book values B_t decrease as t increases (see Figure 6.5.9 in the book)
- The process of finding the values of B_t is called writing down the bond
- Also, P_t form an increasing sequence

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Odds and Ends

- Figure 6.5.11 contains an amortization table for a bond with level coupons, with all the entries in the table described as functions of the inputs of the bond
- In any case, the following recursion formula holds:

$$B_t = (1+j)B_{t-1} - C\,g$$

- *Assignment:* **ALL** examples from Section 6.5.

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