Yield Rates

1. Investment Return

2. Reinvestment Considerations

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An Example: Unknown rate

- At an unknown interest rate i, an investment of 1000 immediately and 1500 at the end of the second year accumulates to 2600 at the end of the fourth year. Find i.

$\Rightarrow$ Set $j = 1 + i$; then, the equation of value at time $\tau = 4$ reads as

$$1000j^4 + 1500j^2 = 2600$$

Solving the quadratic, we get $j^2 = 1.028342$. So, $i = 0.014$.

- The rate we have just calculated is know as the yield rate for an investment.

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- The rate we have just calculated is know as the yield rate for an investment.

The Definition

- Formally, recall the equation of value with reference time τ, i.e.,

$$\sum_{k=1}^{n} C_{t_k}(1+i)^{\tau-t_k} = B(1+i)^{\tau-T}$$

- The rate of interest i which satisfies the above equation (for all other ingredients given and fixed) is called the (annual) yield rate or internal rate of return for that investment
- Alternatively, it may be called the dollar (or money)-weighted yield rate (see SoA Sample Problems for Exam FM: Problem #5)
- The yield rate is often understood as a measure of quality of a certain investment.
- There is a slight problem with this line of thinking, as we can see from the following example ...

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An Example: Non-unique yield rate

- Consider a transaction in which Roger makes payments of \$100 immediately and \$132 at the end of the two years in exchange for a payment in return of \$230 at the end of one year. What is the yield rate of this investment scheme?

⇒ The equation of value is

$$100(1+i)^2 + 132 = 230(1+i)$$

which yields the quadratic equation

$$(1+i)^2 - 2.3(1+i) + 1.32 = 0$$

Now, we factor and obtain

$$[(1+i) - 1.1] \cdot [(1+i) - 1.2] = 0$$

- So that both $i = 0.1$ and $i = 0.2$ may be considered as solutions!
- We **cannot** have any preference towards a particular choice of the yield rate above, but - we have just illustrated why it is of interest to show that a yield rate is *unique* in a particular situation ...
- However, this is not the only "degenerate" case ...

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An Example: Undefined yield

- Assume that Roger is able to borrow \$100 from a bank for one year at 8% effective and immediately lend the said \$100 to Harry for one year at 10% effective. What is Roger's yield rate for this combination of transactions?

⇒ Apparently, Roger is able to make a \$2 profit at the end of the one year - without any net investment at time zero. We might say that this means that the yield rate is infinite ...

However, if Roger were able to borrow \$1000 and then lend that amount of money to Harry (both at the same effective interest rates), his profit at the end of the one year would be \$20; this is clearly preferable to the \$2 profit that he generated in the first case and, yet, the yield rate would be (by the same reasoning as above) infinite

It is sensible to wish to be able to compare the above two cases, but this comparison is not allowed through the definition of yield rates ...

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An Example: Undefined yield

- What is the yield rate on a transaction in which a person makes a payment of \$100 immediately and \$101 at the end of two years, in exchange for a payment of \$200 at the end of one year?

⇒ We need to find i from the equation of value

$$100(1+i)^2 + 101 = 200(1+i)$$

We get

$$100i^2 = -1$$

All the possible solutions are imaginary numbers, and we do not want to accept them as yield rates (there is **no ordering** on the complex numbers that would be suitable for our purposes)

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A sufficient condition for uniqueness of Yield rates

- Hopefully, we have managed to illustrate the importance of showing the **existence and uniqueness** of yield rates . . .
- An easily verified condition for uniqueness of yield rates is the following:

Suppose that the contributions take place at times $t_1 < t_2 < \cdots < t_n$ and that there **exists** a yield rate $i > -1$ such that the outstanding balances

$$B_{t_k}(i) = \sum_{l=1}^{k} C_{t_l}(1+i)^{t_k - t_l}, \text{ for } k \leq n-1$$

are all **of the same sign**.
Then i is the unique yield rate greater than -1.

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An Example: "Bottom line approach"

- Roger's friend Harry wants to open a pizza parlor. So, Roger lends him \$50,000. Harry should return \$55,000 in one year. However, in a year Casper - another friend of Roger's - decides to enter a business venture himself and seeks financial assistance from Roger.
- Then, Roger asks Harry to give Casper \$40, 000 and gets the remaining \$15,000.
- In another year, Casper pays Roger \$45,000. What is Roger's yield rate in this arrangement?

⇒ We are asked only about Roger's yield rate. So, we are going to only look at transactions from his perspective - ignoring all the cash flows that occur between Harry and Casper.

Then the time 2 equation of value, when we concentrate on Roger becomes

$$50000(1+i)^2 = 15000(1+i)^1 + 45000$$

and so $i = \frac{1}{20}(3+\sqrt{369}) - 1$ (where we kept only the positive yield rate)

- We used the "bottom line approach" - focusing on a single investor and only on the transactions that involve that investor

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1 Investment Return

2 Reinvestment Considerations

An Example

- Assume that Roger deposits \$100 at the beginning of each year for 3 years to an account with the effective annual rate of 5%.
  He also seizes the opportunity to invest the *interest payments* from that account at an annual effective rate of 10%. Find the accumulated value of all Roger's accounts a year after his last deposit.

⇒ A time line is really important here!
Altogether, Roger ends up with

$$100 + 100 + 100 = 300$$

on his primary account
On the other hand, his interest payments from his primary account had the nominal values of \$5, \$10 and \$15 and have grown to

$$5(1 + 0.10)^2 + 10(1 + 0.10) + 15 = 32.05$$

The total amount of money that Roger has at the end of the 3 years is 332.05

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  Problems 2.5. 1,2,3
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