Loan Repayment Methods

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2. The Sinking Fund Method

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The Set-up

- When a loan is an amortized loan, each payment is understood to consist of:

1. the interest due on the **outstanding loan balance**;
2. the rest of the payment which goes towards reducing the outstanding loan balance and which is referred to as the principal payment.

- The chart (table) containing the payment amount, interest paid in each payment, principal repaid in each payment and the outstanding balance after each payment is called the amortization schedule

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An Example

- Consider a loan for $1,000 which is to be repaid in four annual payments under the effective annual interest rate of 8%.

  We assume that all payments are equal and get their value as

$$\frac{1000}{a_{\overline{4}|}} = \frac{1000}{3.3121} = 301.92$$

Year #1 Then, the amount of interest contained in the first payment is

$$I_1 = i \cdot 1000 = 0.08 \cdot 1000 = 80$$

Hence, the portion of the first payment that goes toward the reduction of the outstanding balance equals

$$301.92 - 80 = 221.92$$

The outstanding balance at the end of the first year is, then

$$1000 - 221.92 = 778.08$$

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An Example: The amortization schedule

- If we continue the procedure we completed for the first year for the remaining 3 payments, we get the entire amortization schedule:

Year	Pmt	Interest	Principal repaid	OLB
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1	301.29	80.00	221.92	778.08
2	301.29	62.25	239.67	538.41
3	301.29	43.07	258.85	279.56
4	301.29	22.36	279.56	0

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An Example: Smaller final payment

- A \$1,000 loan is being repaid by payments of \$100 (plus the final smaller payment) at the end of each quarter-year for as long as it is necessary. Assume that $i^{(4)} = 0.16$.

  Find the amount of interest and the amount of principal repaid in the fourth payment.

⇒ Using the retrospective method (why??), we get that the outstanding loan balance at the beginning of the fourth quarter equals

$$1000(1.04)^3 - 100 \cdot s_{\overline{3}|} = 1124.86 - 312.16 = 812.70$$

The interest that is to be repaid in the fourth payment is exactly the amount of interest that is accrued during the fourth quarter-year on the balance above, i.e.,

$$0.04 \cdot 812.70 = 32.51$$

Evidently, the fourth payment is not yet the final, smaller one. So, the principal payment contained in the fourth payment is

$$100 - 32.51 = 67.49$$

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- Hence, a single "lump-sum" payment should repay the entire loan at the end of the loan term.
- In order to finance this final payment, the borrower might wish to make deposits on a separate savings account during the life of the loan. This account is called the sinking fund account.
- This repayment method is referred to as the sinking fund method
- Note that we need to differentiate between two accounts in this repayment schedule, i.e., there are **two interest rates** at play
- We **usually** denote the interest rate governing the loan by i, and the interest rate of the sinking fund account by j
- It is customary (but not necessary) that we assume that $j < i$

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Some more notation

- Assume that the loan amount is denoted by L.
- Then, at the end of each period, one needs to pay the interest payment $L \cdot i$
  and the sinking fund deposit of
  $$\frac{L}{s_{\overline{n|}\,j}}$$
- So, the total payment at the end of each period is
  $$L \cdot \left(i + \frac{1}{s_{\overline{n|}\,j}} \right)$$
- We define
  $$a_{\overline{n|}\,i\&j} = \frac{1}{i + \frac{1}{s_{\overline{n|}\,j}}} = \frac{a_{\overline{n|}\,j}}{(i-j)a_{\overline{n|}\,j} + 1}$$
- Note that if $i = j$, then we are back in the amortized loan setting!

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