Premium-Discount Formula and Other Bond Pricing Formulas

1. Premium-Discount Formula

2. Other Pricing Formulas for Bonds
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The formula and selling at a premium

Assignment: All the examples in section 6.2!

• The premium-discount pricing formula for bonds reads as

\[ P = C(g - j)a_{mj} + C \]

where \( C \) is the redemption amount, \( g \) is the modified coupon rate, \( j \) is the effective yield rate per coupon period, and \( n \) is the number of coupons.

• If \( P > C \), we say that the bond sells at a premium

• The value \( P - C \) is called the premium or amount of premium for the bond, i.e.,

\[ P - C = C(g - j)a_{mj} \]

• So, the bond sells at a premium iff \( g > j \)
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Selling at a discount

• If $P < C$, we say that the bond sells at a discount
  
• Then, the value $C - P$ is called the discount or amount of discount on the bond and it equals

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**An Example**

- Find the price of a $1,000 par value 10-year bond with coupons at 8.4% convertible semiannually, which will be redeemed at $1,050. The bond is bought to yield 10% convertible semiannually.

⇒ In this example, the parameters are:

\[
F = 1000 \\
C = 1050 \\
r = \frac{0.084}{2} = 0.042 \\
g = \frac{1000}{1050} \cdot 0.042 = 0.04 \\
j = \frac{0.1}{2} = 0.05 \\
n = 20 \\
K = 1050 \cdot 1.05^{-20} = 395.7340 \\
G = \frac{0.042}{0.05} \cdot 1000 = 840
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\[ G = \frac{0.042}{0.05} \cdot 1000 = 840 \]
An Example (cont’d)

Using the **basic pricing formula**, we get

\[
P = F r a_{\overline{m}} + K
\]

\[
= 1000 \cdot 0.042 \cdot a_{20|0.05} + 395.7340
\]

\[
= 42 \cdot 12.4622 + 395.7340
\]

\[
= 919.15
\]

Using the **premium-discount formula**, we get

\[
P = C + (F r - Cj)a_{\overline{m}}
\]

\[
= 1050 + (42 - 52.50)a_{20|0.05}
\]

\[
= 1050 + (-9.50) \cdot 12.4622
\]

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= 919.15
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Of course, the two prices are the same

• **Assignment**: Examples 6.3.5 and 6.3.6
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\[ P = F r \overline{a}_{\text{m}} + K \]
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An Example (cont’d)

Using the **basic pricing formula**, we get

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P = Fra_m + K \\
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2 Other Pricing Formulas for Bonds
The Base Amount Formula

• If we substitute the expression for the value of the annuity in the basic formula, we get

\[ P = G - Gv_j^n + Cv_j^n = (C - G)v_j^n + G \]

where \( G \) denotes the base amount, \( v_j \) is the discount factor per coupon period and \( n \) is the number of coupons

• The above formula is referred to as the base amount formula
The Base Amount Formula

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Reconsidering the earlier example, we can reevaluate the price of the bond using the base amount formula as

\[ P = G + (C - G)v_j^n \]

\[ = 840 + (1050 - 840) \left( \frac{1}{1.05} \right)^{20} \]

\[ = 840 + 210 \cdot 0.37689 \]

\[ = 919.15 \]
If we do not know the number of coupons \( n \), but we know the present value \( K \) of the redemption amount, we use Makeham’s formula:

\[
P = K + \frac{g}{j} \cdot (C - K)
\]

where \( g \) stands for the coupon rate, \( j \) is the effective yield rate per coupon period, \( C \) is the redemption amount and \( K \) is the present value of the redemption amount.
If we look at our example again, using Makeham’s formula, we obtain:

\[ P = K + \frac{g}{j}(C - K) \]

\[ = 395.7340 + \frac{0.04}{0.05}(1050 - 395.7340) \]

\[ = 919.15 \]