

Chapter 11

Binomial Option Pricing: II

Question 11.1.

a) Early exercise occurs only at strike prices of 70 and 80. The value of the one period binomial European 70 strike call is \$23.24, while the value of immediate exercise is $100 - 70 = 30$. The value of the 80 strike European call is \$19.98, while the value of immediate exercise is \$20.

b) From put-call-parity, we observe the following:

$$C = Se^{-\delta} - Ke^{-r} + P = 100 \times 0.92311 - K + P = 92.31164 - K + P$$

Clearly, as long as $100 - K$ is larger than $92.31164 - K + P$ or $P < 7.688$, we will exercise the option early. Already at a strike of 90, $100 - K = 10$ is smaller than the continuation value given by the above formula, so we do not exercise early.

c) The value of a put falls when the strike price decreases. From part a), we learned that the decisive criterion was that $P < 7.688$. Therefore, once we cross this threshold, all other calls will be exercised as well.

Question 11.2.

By introducing a non-zero interest rate, we increase the cost of early exercise, because we pay the strike before expiration, and lose interest on it. We see that we only exercise the call with a strike of 70. The value of the European 70-strike call is \$27.69, the value of immediate exercise is \$30.

The decisive condition, derived from put-call-parity, is now:

$$C = Se^{-\delta} - Ke^{-r} + P = 100 \times 0.92311 - K \times 0.92311 + P = 92.31164 - 0.92311K + P$$

Therefore, we will exercise whenever

$$\begin{aligned} 100 - K &> 92.31164 - 0.92311K + P \\ \Leftrightarrow P &< 7.688 - 0.07688K \end{aligned}$$

This condition is indeed fulfilled at a strike price of 70. Clearly, this boundary is attained earlier than the boundary of exercise 11.1., so we will stop early exercise at lower strike prices when the interest rates are high.

Question 11.3.

If the dividend yield is equal to zero, there is no benefit to early exercise the call option. Only costs are associated with it (losing interest on the strike, losing insurance). It is never optimal to early exercise a call option that does not pay a dividend.

Question 11.4.

a) Early exercise occurs only at a strike price of 130. The value of the one period binomial European 130 strike put is \$26.38, while the value of immediate exercise is $130 - 100 = 30$.

b) From put-call-parity, we observe the following:

$$P = Ke^{-r} - Se^{-\delta} + C = K \times 0.9231164 - S + C = 92.31164K - S + C$$

Clearly, as long as $K - 100$ is larger than $92.31164K - 100 + C$ or $C < 0.07688K$, we will exercise the option early. Already at a strike of 120, $0.07688 \times 120 = 9.2256$ is smaller than the value of the European call option with a strike of 120 (with a price of \$10.30), which means that we do not exercise prior to expiration.

c) The value of a call falls when the strike price increases. From part b), we learned that the decisive criterion was that $C < 0.07688K$. Therefore, if this criterion is fulfilled for some threshold $K(*)$, it is fulfilled for every K above $K(*)$.

Question 11.5.

By introducing a non-zero dividend yield, we increase the cost of early exercise, because now giving up the stock early via exercise of the put option means that we lose dividend income. We see that we only exercise the put with a strike of 130, and just barely: The value of the European 130-strike put is \$29.65, the value of immediate exercise is \$30.

The decisive condition, derived from put-call-parity, is now:

$$P = Ke^{-r} - Se^{-\delta} + C = K \times 0.92311 - S \times 0.92311 + C = 0.92311K - 92.31164 + C$$

Therefore, we will exercise whenever

$$\begin{aligned} K - 100 &> 0.92311K - 92.31164 + C \\ \Leftrightarrow C &< 0.07688K - 7.688 \end{aligned}$$

This condition is indeed fulfilled at a strike price of 130. Clearly, this boundary is attained earlier than the boundary of exercise 11.4., so we will stop early exercise earlier when the dividend yield is high.

Question 11.6.

We now have from put-call-parity:

$$P = Ke^{-r} - Se^{-\delta} + C = K - S \times 0.92311 + C = 0.92311K - 92.31164 + C$$

We would exercise early if:

$$\begin{aligned} K - 100 &> K - 92.31164 + C \\ \Leftrightarrow C &< -7.688, \end{aligned}$$

which can never be true. It is never optimal to early exercise, because the sole advantage of early exercise, receiving the interest on the strike earlier, has been removed.

Question 11.7.

a) We need to find the true probabilities for the stock going up. We will use formula (11.4) of the main text. To find u and d , note that $h = 1/10 = 0.1$.

$$u = e^{(r-\delta)h+\sigma\sqrt{h}} = e^{(0.08)\times 0.1+0.3\times\sqrt{0.1}} = 1.10835$$

$$d = e^{(r-\delta)h-\sigma\sqrt{h}} = e^{(0.08)\times 0.1-0.3\times\sqrt{0.1}} = 0.91680$$

$$p = \frac{e^{ah} - d}{u - d} = \frac{1.015113 - 0.91680}{1.10835 - 0.91680} = 0.51325$$

Now, we can calculate the binomial tree (as in figure 11.4) and back out the option prices and the relevant discount rates (= required expected returns of the option holder). We obtain for the European Call (the first entry is the stock price, the second the option price and the third is the required rate of return):

										279.74
										179.74
									252.39	
									153.19	
								227.72	0.20	231.39
								129.31		131.39
							205.46	0.20	208.77	
							107.83		109.57	
						185.37	0.21	188.36	0.21	191.40
						88.52		89.95		91.40
					167.25	0.23	169.95	0.23	172.69	
					71.17		72.32		73.49	
				150.90	0.24	153.34	0.24	155.81	0.24	158.32
				55.79		56.49		57.40		58.32
			136.15	0.27	138.35	0.27	140.58	0.27	142.85	
			42.54		42.66		42.95		43.64	
		122.84	0.29	124.82	0.30	126.84	0.31	128.88	0.31	130.96
		31.52		31.13		30.74		30.47		30.96
	110.83	0.32	112.62	0.33	114.44	0.35	116.28	0.37	118.16	
	22.73		21.99		21.13		20.11		18.96	
100.00	0.34	101.61	0.36	103.25	0.39	104.92	0.43	106.61	0.51	108.33
15.96		15.07		14.01		12.71		11.00		8.33
0.37	91.68	0.39	93.16	0.42	94.66	0.47	96.19	0.57	97.74	
	10.06		9.01		7.75		6.16		3.94	
	0.42	84.05	0.46	85.41	0.51	86.78	0.61	88.18	0.83	89.61
		5.65		4.60		3.37		1.86		0.00
		0.48	77.06	0.54	78.30	0.64	79.56	0.83	80.85	
			2.67		1.81		0.88		0.00	
			0.57	70.65	0.66	71.79	0.83	72.94	0.00	74.12
				0.96		0.42		0.00		0.00
				0.68	64.77	0.83	65.81	0.00	66.88	
					0.20		0.00		0.00	
					0.83	59.38	0.00	60.34	0.00	61.31
						0.00		0.00		0.00
						0.00	54.44	0.00	55.32	
							0.00		0.00	
							0.00	49.91	0.00	50.72
								0.00		0.00
								0.00	45.76	
									0.00	
									0.00	41.95
										0.00

Note that the price is identical to the price we get via risk-neutral pricing with the BinomCall function.

For the European Put option, we have:

										279.74
										0.00
									252.39	
									0.00	
								227.72	0.00	231.39
								0.00		0.00
							205.46	0.00	208.77	
							0.00		0.00	
						185.37	0.00	188.36	0.00	191.40
						0.00		0.00		0.00
					167.25	0.00	169.95	0.00	172.69	
					0.00		0.00		0.00	
				150.90	0.00	153.34	0.00	155.81	0.00	158.32
				0.20		0.00		0.00		0.00
			136.15	-0.65	138.35	0.00	140.58	0.00	142.85	
			0.94		0.39		0.00		0.00	
		122.84	-0.49	124.82	-0.65	126.84	0.00	128.88	0.00	130.96
		2.48		1.62		0.76		0.00		0.00
	110.83	-0.37	112.62	-0.47	114.44	-0.65	116.28	0.00	118.16	
	4.94		3.92		2.77		1.46		0.00	
100.00	-0.28	101.61	-0.34	103.25	-0.45	104.92	-0.65	106.61	0.00	108.33
8.27		7.26		6.07		4.64		2.81		0.00
-0.21	91.68	-0.25	93.16	-0.31	94.66	-0.42	96.19	-0.65	97.74	
	11.43		10.41		9.17		7.61		5.40	
	-0.18	84.05	-0.22	85.41	-0.28	86.78	-0.38	88.18	-0.65	89.61
		15.39		14.51		13.43		12.09		10.39
		-0.16	77.06	-0.19	78.30	-0.23	79.56	-0.32	80.85	
			20.17		19.59		18.94		18.36	
			-0.12	70.65	-0.15	71.79	-0.18	72.94	-0.23	74.12
				25.62		25.48		25.47		25.88
				-0.09	64.77	-0.11	65.81	-0.12	66.88	
					31.51		31.81		32.33	
					-0.06	59.38	-0.07	60.34	-0.07	61.31
						37.47		38.07		38.69
						-0.03	54.44	-0.03	55.32	
							43.19		43.89	
							-0.01	49.91	-0.01	50.72
								48.50		49.28
								0.01	45.76	
									53.45	
									0.02	41.95
										58.05

b) If we increase the standard deviation of the stock return, we decrease the required return of the call option and increase the required (negative) return of the put option. The following table shows the required returns for the first two of the 10 nodes:

For the call option:		and for the put option:	
	118.07		118.07
	34.84		9.82
100.00	0.26	100.00	-0.12
23.28		15.59	
0.28	86.06	-0.07	86.06
	13.75		20.74
	0.31		-0.06

Question 11.8.

For the following questions, we will report the first two of the 10 nodes. We have for the European call options of strike 70, 80, 90 and 100:

Delta and B:

$K = 70$		$K = 80$		$K = 90$		$K = 100$	
	0.98		0.94		0.88		0.77
	-62.84		-66.81		-68.35		-63.16
0.95		0.88		0.79		0.66	
-58.76		-59.44		-57.40		-50.19	
	0.91		0.81		0.68		0.54
	-55.96		-53.66		-48.32		-39.16

Call option price and gamma, the required rate of return:

$K = 70$		$K = 80$		$K = 90$		$K = 100$	
	110.83		110.83		110.83		110.83
	45.95		37.38		29.21		22.73
100.00	0.24	100.00	0.27	100.00	0.31	100.00	0.34
36.14		28.34		21.16		15.96	
0.26	91.68	0.30	91.68	0.34	91.68	0.376	91.68
	27.77		20.56		14.17		10.06
	0.29		0.33		0.39		0.42

Part 3 Options

Delta and B:

$K = 110$		$K = 120$		$K = 130$	
	0.67		0.55		0.43
	-57.32		-48.16		-39.00
0.54		0.43		0.31	
-42.79		-34.46		-26.13	
	0.40		0.29		0.19
	-30.24		-22.54		-14.83

Call option price and gamma:

$K = 110$		$K = 120$		$K = 130$	
	110.83		110.83		110.83
	16.63		12.52		8.41
100.00	0.39	100.00	0.41	100.00	0.47
11.14		8.18		5.22	
0.41	91.68	0.44	91.68	0.49	91.68
	6.30		4.36		2.41
	0.48		0.50		0.57

We clearly see that the more the option is out-of-the-money, the higher the required expected return is. This is a consequence of the option becoming more and more leveraged as it moves out of the money.

Question 11.9.

We have for a time to expiration of 3 months:

Call option price and gamma, the required rate of return:

$K = 70$		$K = 80$		$K = 90$		$K = 100$	
	105.07		105.07		105.07		105.07
	36.32		26.58		17.43		9.85
100.00	0.28	100.00	0.35	100.00	0.46	100.00	0.61
31.40		21.88		13.41		7.01	
0.30	95.56	0.39	95.56	0.51	95.56	0.66	95.56
	26.83		17.48		9.63		4.32
	0.33		0.43		0.59		0.77

$K = 110$		$K = 120$		$K = 130$	
105.07		105.07		105.07	
4.73		1.95		0.67	
100.00	0.77	100.00	0.94	100.00	1.11
3.11		1.19		0.39	
0.82	95.56	0.98	95.56	1.14	95.56
1.58		0.48		0.11	
0.96		1.14		1.33	

We have for a time to expiration of 6 months:

Call option price and gamma, the required rate of return:

$K = 70$		$K = 80$		$K = 90$		$K = 100$	
107.37		107.37		107.37		107.37	
39.87		30.50		21.89		14.84	
100.00	0.27	100.00	0.32	100.00	0.38	100.00	0.45
32.90		23.99		16.31		10.52	
0.29	93.89	0.35	93.89	0.42	93.89	0.49	93.89
26.63		18.09		11.23		6.55	
0.32		0.39		0.48		0.56	

$K = 110$		$K = 120$		$K = 130$	
107.37		107.37		107.37	
9.42		5.22		3.08	
100.00	0.53	100.00	0.63	100.00	0.68
6.33		3.27		1.87	
0.56	93.89	0.67	93.89	0.71	93.89
3.48		1.48		0.76	
0.66		0.78		0.83	

We have for a time to expiration of 1 year:

Call option price and gamma, the required rate of return:

$K = 70$		$K = 80$		$K = 90$		$K = 100$	
110.83		110.83		110.83		110.83	
45.95		37.38		29.21		22.73	
100.00	0.24	100.00	0.27	100.00	0.31	100.00	0.34
36.14		28.34		21.16		15.96	
0.26	91.68	0.30	91.68	0.34	91.68	0.376	91.68
27.77		20.56		14.17		10.06	
0.29		0.33		0.39		0.42	

	$K = 110$		$K = 120$		$K = 130$	
	110.83		110.83		110.83	
	16.63		12.52		8.41	
100.00	0.39	100.00	0.41	100.00	0.47	
11.14		8.18		5.22		
0.41	91.68	0.44	91.68	0.49	91.68	
	6.30		4.36		2.41	
	0.48		0.50		0.57	

We have for a time to expiration of 2 years:

Call option price and gamma, the required rate of return:

	$K = 70$		$K = 80$		$K = 90$		$K = 100$
	116.20		116.20		116.20		116.20
	56.21		48.66		41.15		35.30
100.00	0.22	100.00	0.23	100.00	0.25	100.00	0.27
41.97		35.42		28.93		24.35	
0.23	88.86	0.25	88.86	0.27	88.86	0.29	88.86
	30.77		24.92		19.13		15.51
	0.26		0.28		0.31		0.32

	$K = 110$		$K = 120$		$K = 130$	
	116.20		116.20		116.20	
	29.44		24.18		20.56	
100.00	0.29	100.00	0.31	100.00	0.32	
19.76		15.71		13.18		
0.30	88.86	0.33	88.86	0.34	88.86	
	11.89		8.78		7.12	
	0.34		0.37		0.38	

We can clearly observe the time to maturity effect in expected returns: The shorter the time to maturity is, the more speculative the options are, and the higher the required rate of return is. Thus, we have two effects: A moneyness effect, and a time to maturity effect.

Question 11.10.

For the following questions, we will, due to space limitations, only report the first two of the 10 nodes. We have for the European put options of strike 70, 80, 90 and 100:

Delta and B:

	$K = 70$		$K = 80$		$K = 90$		$K = 100$	
	-0.02		-0.06		-0.12		-0.23	
	2.29		7.63		15.40		29.89	
-0.05			-0.12		-0.21		-0.34	
5.85			14.41		25.68		42.12	
	-0.09		-0.19		-0.32		-0.46	
	9.18		20.79		35.43		53.90	

Put option price and gamma, the required rate of return:

	$K = 70$		$K = 80$		$K = 90$		$K = 100$	
	110.83		110.83		110.83		110.83	
	0.25		0.99		2.13		4.94	
100.00	-0.51	100.00	-0.41	100.00	-0.37	100.00	-0.28	
0.75	2.19		4.24		8.27			
-0.41	91.68	-0.32	91.68	-0.28	91.68	-0.21	91.68	
	1.23		3.32		6.23		11.43	
	-0.39		-0.30		-0.25		-0.18	

Delta and B:

	$K = 110$		$K = 120$		$K = 130$	
	-0.33		-0.45		-0.57	
	45.04		63.51		81.97	
-0.46			-0.57		-0.69	
58.75			76.31		93.87	
	-0.60		-0.71		-0.81	
	72.12		89.13		106.14	

Call option price and gamma:

	$K = 110$		$K = 120$		$K = 130$	
	110.83		110.83		110.83	
	8.16		13.35		18.55	
100.00	-0.24	100.00	-0.19	100.00	-0.16	
12.68	18.95		25.23			
-0.18	91.68	-0.13	91.68	-0.11	91.68	
	16.98		24.34		31.69	
	-0.15		-0.11		-0.09	

We clearly see that the more the put option is out-of-the-money, the lower the required expected return is.

Question 11.11.

We have for a time to expiration of 3 months:

Put option price and gamma, the required rate of return:

$K = 70$		$K = 80$		$K = 90$		$K = 100$	
105.07		105.07		105.07		105.07	
0.00		0.09		0.75		3.00	
100.00	-1.39	100.00	-1.14	100.00	-0.88	100.00	-0.64
0.01		0.30		1.63		5.03	
-1.32	95.56	-0.94	95.56	-0.70	95.56	-0.51	95.56
0.02		0.50		2.47		6.98	
-1.32		-0.91		-0.65		-0.45	

$K = 110$		$K = 120$		$K = 130$	
105.07		105.07		105.07	
7.70		14.74		23.29	
100.00	-0.45	100.00	-0.31	100.00	-0.21
10.93		18.82		27.81	
-0.35	95.56	-0.24	95.56	-0.16	95.56
14.05		22.78		32.23	
-0.30		-0.19		-0.12	

We have for a time to expiration of 6 months:

Put option price and gamma, the required rate of return:

$K = 70$		$K = 80$		$K = 90$		$K = 100$	
107.37		107.37		107.37		107.37	
0.03		0.30		1.34		3.94	
100.00	-0.89	100.00	-0.73	100.00	-0.58	100.00	-0.43
0.15		0.86		2.78		6.60	
-0.74	93.89	-0.58	93.89	-0.45	93.89	-0.33	93.89
0.27		1.38		4.16		9.13	
-0.72		-0.56		-0.42		-0.29	

$K = 110$			$K = 120$			$K = 130$		
107.37			107.37			107.37		
8.17			13.61			21.11		
100.00	−0.33		100.00	−0.26		100.00	−0.19	
12.01			18.57			26.77		
−0.25	93.89		−0.19	93.89		−0.14	93.89	
15.70			23.35			32.27		
−0.21			−0.16			−0.11		

We have for a time to expiration of 1 year:

Put option price and gamma, the required rate of return:

$K = 70$				$K = 80$				$K = 90$				$K = 100$			
110.83				110.83				110.83				110.83			
0.25				0.99				2.13				4.94			
100.00	−0.51			100.00	−0.41			100.00	−0.37			100.00	−0.28		
0.75				2.19				4.24				8.27			
−0.41	91.68			−0.32	91.68			−0.28	91.68			−0.21	91.68		
1.23				3.32				6.23				11.43			
−0.39				−0.30				−0.25				−0.18			

$K = 110$			$K = 120$			$K = 130$		
110.83			110.83			110.83		
8.16			13.35			18.55		
100.00	−0.24		100.00	−0.19		100.00	−0.16	
12.68			18.95			25.23		
−0.18	91.68		−0.13	91.68		−0.11	91.68	
16.98			24.34			31.69		
−0.15			−0.11			−0.09		

We have for a time to expiration of 2 years:

Put option price and gamma, the required rate of return:

$K = 70$				$K = 80$				$K = 90$				$K = 100$			
116.20				116.20				116.20				116.20			
0.62				1.73				2.88				5.68			
100.00	−0.30			100.00	−0.25			100.00	−0.23			100.00	−0.18		
1.62				3.59				5.63				9.56			
−0.23	88.86			−0.19	88.86			−0.17	88.86			−0.13	88.86		
2.53				5.34				8.20				13.24			
−0.22				−0.17				−0.15				−0.11			

	$K = 110$		$K = 120$		$K = 130$	
	116.20		116.20		116.20	
	8.49		11.89		16.92	
100.00	−0.16	100.00	−0.14	100.00	−0.11	
13.49		17.97		23.95		
−0.11	88.86	−0.09	88.86	−0.07	88.86	
	18.28		23.84		30.83	
	−0.09		−0.07		−0.05	

We can clearly observe the time to maturity effect in expected returns: The shorter the time to maturity, the smaller the expected return of the put options is.

Question 11.12.

For $n = 3$, u and d are calculated as follows:

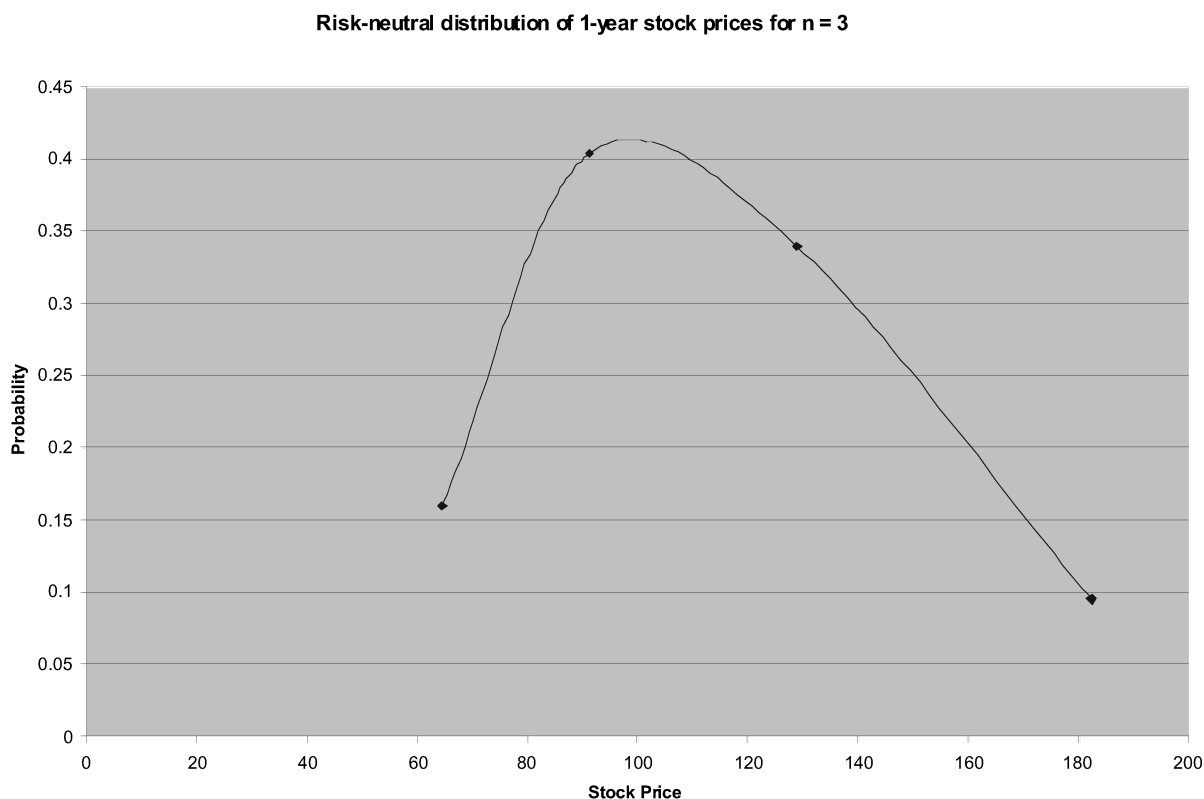
$$u = e^{(r-\delta)h+\sigma\sqrt{h}} = e^{(0.08)\times 1/3+0.3\times\sqrt{1/3}} = 1.2212$$

$$d = e^{(r-\delta)h-\sigma\sqrt{h}} = e^{(0.08)\times 1/3-0.3\times\sqrt{1/3}} = 0.8637$$

Now we can calculate $p^* = \frac{e^{(r-\delta)h} - d}{u - d} = 0.4568$

$n - i$	Stock price	Probability
0	182.141786	0.09532291
1	128.814742	0.34004825
2	91.1006651	0.40435476
3	64.4284267	0.16027409

The figure looks like:



for $n = 10$, we have:

$$u = e^{(r-\delta)h + \sigma\sqrt{h}} = e^{(0.08) \times 1/10 + 0.3 \times \sqrt{1/10}} = 1.1083$$

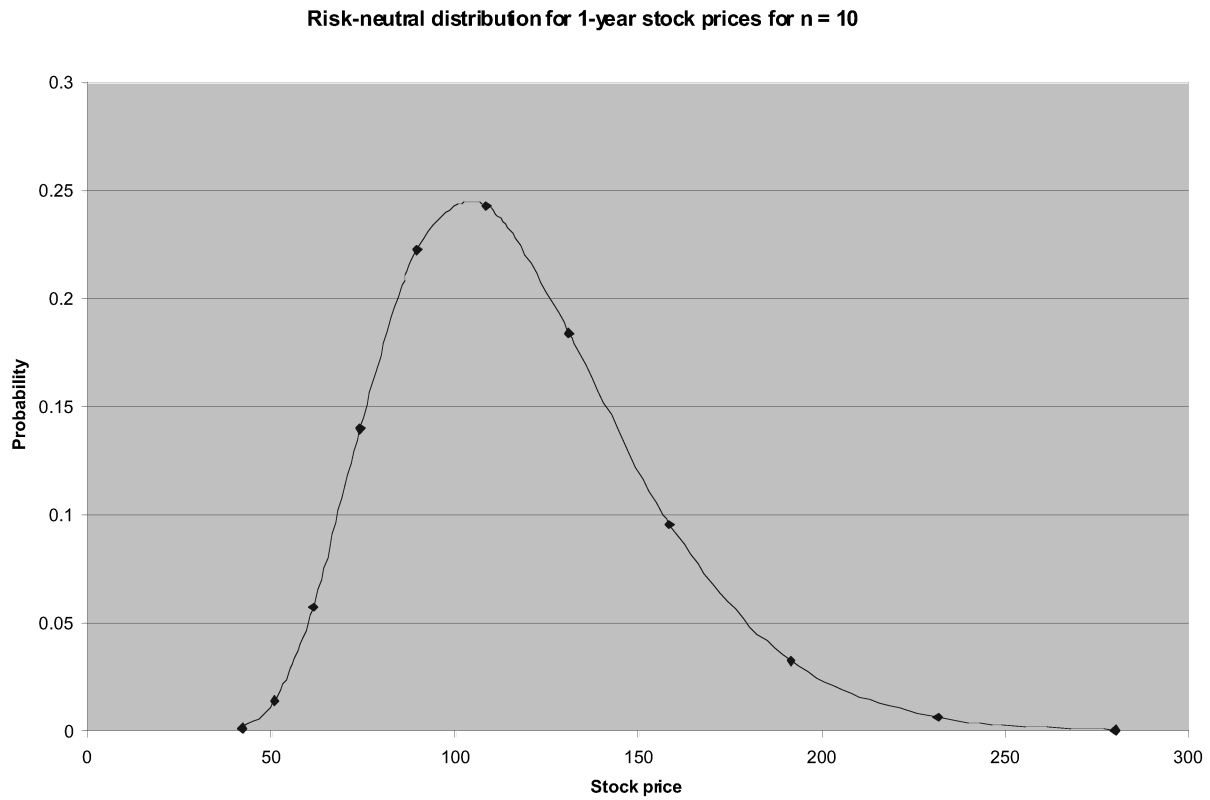
$$d = e^{(r-\delta)h - \sigma\sqrt{h}} = e^{(0.08) \times 1/10 - 0.3 \times \sqrt{1/10}} = 0.9168$$

Now we can calculate $p^* = \frac{e^{(r-\delta)h} - d}{u - d} = 0.4763$

$n - i$	Stock price	Probability
0	279.738009	0.00060091
1	231.392829	0.00660713
2	191.402811	0.03269085
3	158.32399	0.0958508
4	130.961953	0.18443127
5	108.328707	0.24334174
6	89.6070076	0.22296472
7	74.1208498	0.14008734
8	61.3110573	0.0577605
9	50.7150924	0.014113
10	41.9503547	0.00155174

Part 3 Options

The figure looks like:



Question 11.13.

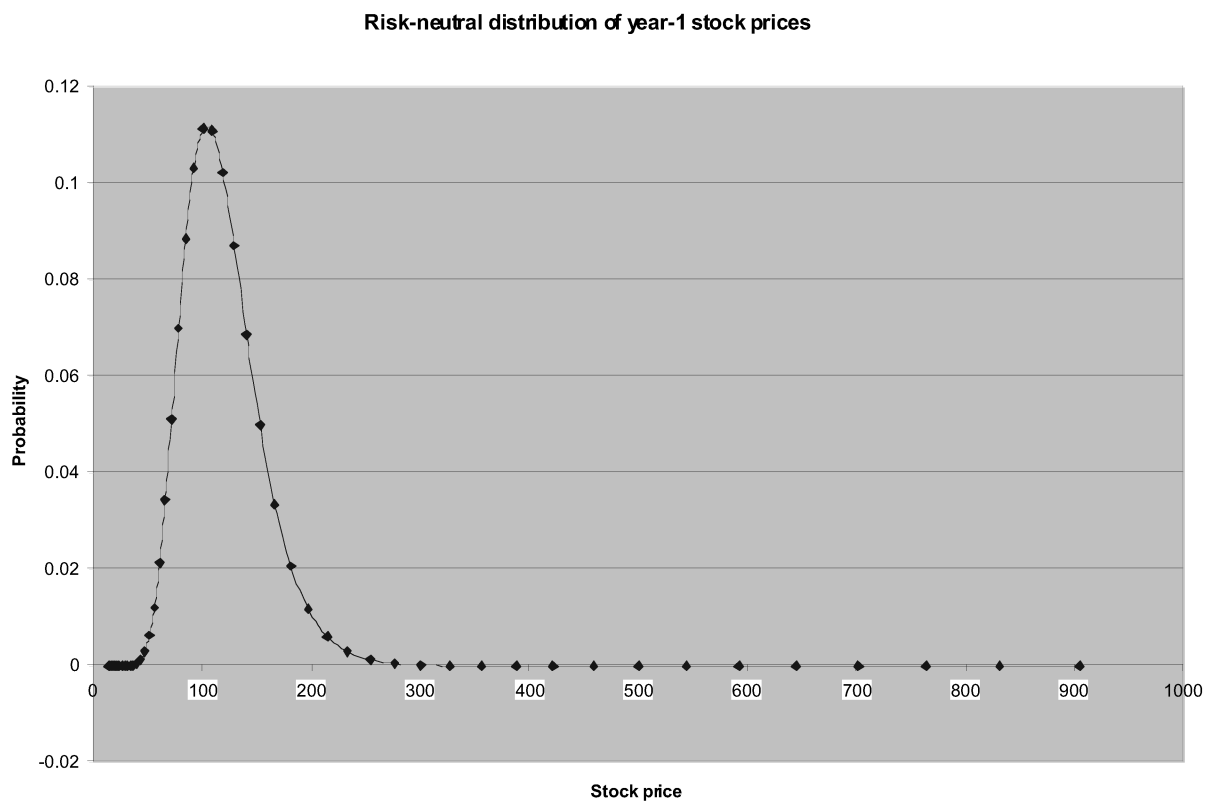
For $n = 50$, we have $h = 0.02$:

$$u = e^{(r-\delta)h + \sigma\sqrt{h}} = e^{(0.08) \times 0.02 + 0.3 \times \sqrt{0.02}} = 1.0450$$

$$d = e^{(r-\delta)h - \sigma\sqrt{h}} = e^{(0.08) \times 0.02 - 0.3 \times \sqrt{0.02}} = 0.9600$$

Now we can calculate $p^* = \frac{e^{(r-\delta)h} - d}{u - d} = 0.4894$

We get the following diagram:



To obtain the required probabilities, we sum all probabilities for which the final stock price is below 80 or above 120 respectively. We obtain:

$$\Pr(S < 80) = 0.2006$$

$$\Pr(S > 120) = 0.2829$$

Question 11.14.

a) We can calculate the forward prices as:

$$F_{0,4 \text{ months}} = 100e^{0.08 \times 1/3} = 102.7025$$

$$F_{0,8 \text{ months}} = 100e^{0.08 \times 2/3} = 105.4781$$

$$F_{0,1 \text{ year}} = 100e^{0.08} = 108.3287$$

b) After $t = 1/3$ years, we have:

Stock price	Probability
122.124611	0.45680665
86.3692561	0.54319335

Part 3 Options

This yields an expectation of 102.70254, equivalent to the forward price of part a).

After $t = 2/3$ years, we have:

Stock price	Probability
149.144206	0.20867232
105.478118	0.49626867
74.596484	0.29505901

This yields an expectation of 105.4781, equivalent to the forward price of part a).

After 1 year, we have:

Stock price	Probability
182.141781	0.0953229
128.814741	0.34004825
91.1006658	0.40435476
64.4284283	0.16027409

This yields an expectation of 108.3287, equivalent to the forward price of part a).

Question 11.15.

We have to be careful to adapt the time period h . h is now $1/50$. This yields:

$u =$	1.04500995
$d =$	0.95999576
p^*	0.48939499

Now we can proceed as in exercise 11.14 to calculate the expected forward price. The result is, of course, the same as in exercise 11.14 for the one year forward: $F_{0,1year} = 108.3287$.

Question 11.16.

Stock Tree								233.621
								210.110
								188.966
								169.949
								152.847
								137.465
								123.631
								111.190
100.000	111.190	95.000	100.000	100.000	100.000	100.000	100.000	100.000
	89.937		89.937	89.937	89.937	89.937	89.937	89.937
		80.886	80.886	80.886	80.886	80.886	80.886	80.886
			72.746	72.746	72.746	72.746	72.746	72.746
				65.425	65.425	65.425	65.425	65.425
					58.841	58.841	58.841	58.841
						52.920	52.920	52.920
							47.594	47.594
								42.804
European Call								138.621
								114.746
								93.966
								74.836
								57.847
								42.554
								28.631
								16.442
15.140	22.118	13.563	11.670	9.221	7.065	5.080	3.099	1.118
	8.703	4.183	2.760	1.182	0.575	0.000	0.000	0.000
			0.280	0.000	0.000	0.000	0.000	0.000
				0.000	0.000	0.000	0.000	0.000
					0.000	0.000	0.000	0.000
						0.000	0.000	0.000
							0.000	0.000
								0.000

American Call								138.621
						115.110		
					93.966			93.966
				74.949		74.949		
			57.981		57.884			57.847
		43.422		42.746		42.554		
	31.482		30.226		28.976			28.631
	22.144	20.578		18.736		16.442		
15.153		13.566	11.670		9.221			5.000
	8.704		7.065	5.080		2.431		
		4.183		2.760	1.182			0.000
			1.483		0.575	0.000		
				0.280		0.000		0.000
					0.000		0.000	
						0.000		0.000
							0.000	
								0.000
European Put								0.000
						0.000		
					0.000			0.000
				0.000		0.000		0.000
			0.458		0.000			0.000
		1.819		0.909		0.000		0.000
	4.265		3.169		1.804		0.000	
7.713		6.711		5.414		3.582		0.000
	11.194		10.263		9.006		7.110	
		15.742		15.146		14.420		14.114
			21.342		21.373		21.762	
				27.745		28.506		29.575
					34.444		35.580	
						40.857		42.080
							46.757	
								52.196

Stock Tree							230.142
						207.371	
					186.852		186.153
				168.363		167.733	
			151.704		151.137		150.571
		136.694		136.182		135.672	
	123.168		122.707		122.248		121.790
	110.981	110.566		110.152		109.740	
100.000		99.626	99.253		98.881		98.511
	89.768		89.432	89.097		88.764	
		80.583		80.281	79.981		79.682
			72.338		72.067	71.797	
				64.936		64.693	64.451
					58.292		58.074
						52.328	52.132
							46.973
							42.167

European Call

								135.142
							112.024	
						91.412		91.153
					73.044		72.634	
				56.684		56.140		55.571
			42.576		41.460		40.772	
		30.969		29.322		27.611		26.790
	21.865		19.981		17.771		15.001	
15.029		13.198		11.040		8.287		3.511
	8.494		6.680		4.529		1.738	
		3.960		2.453		0.861		0.000
			1.319		0.426		0.000	
				0.211		0.000		0.000
					0.000		0.000	
						0.000		0.000
							0.000	
								0.000

American Call

								135.142
							112.371	
						91.852		91.153
					73.363		72.733	
				56.854		56.190		55.571
			42.667		41.485		40.772	
		31.016		29.334		27.611		26.790
	21.890		19.987		17.771		15.001	
15.042		13.201		11.040		8.287		3.511
	8.496		6.680		4.529		1.738	
		3.960		2.453		0.861		0.000
			1.319		0.426		0.000	
				0.211		0.000		0.000
					0.000		0.000	
						0.000		0.000
							0.000	
								0.000

We chose the stock of IBM from June 1st 1997 to May 31st 2002 (<http://finance.yahoo.com>). Please note that we must calculate the continuously compounded returns before we can calculate the weekly standard deviation. (c.f. table 11.1 of the textbook). We obtain the annual standard deviation by multiplying our weekly estimate by the square root of 52.

In particular, this yields:

Period	Weekly	Annual
97/6 – 02/05	0.050	0.360
97/6 – 98/5	0.040	0.287
98/6 – 99/5	0.049	0.355
99/6 – 00/5	0.052	0.376
00/6 – 01/5	0.063	0.457
01/6 – 02/5	0.040	0.287
97/6 – 97/11	0.039	0.279
97/12 – 98/5	0.041	0.298
98/6 – 98/11	0.043	0.311
98/12 – 99/5	0.056	0.401
99/6 – 99/11	0.055	0.395
99/12 – 00/5	0.050	0.364
00/6 – 00/11	0.060	0.432
00/12 – 01/5	0.067	0.486
01/6 – 01/11	0.037	0.268
01/12 – 02/5	0.041	0.297

We can observe a time trend in the volatility estimate: Volatility is rising throughout the nineties, up to a record level of over 45 percent in 2000/2001. Only the last year of data shows a reversal in the volatility.

There does not seem to be an additional pattern to be detected when we conduct the semiannual volatility estimates.

Question 11.19.

We chose the stocks of IBM, General Electric (ticker symbol GE) and Exxon (ticker symbol XOM) from <http://finance.yahoo.com>.

We obtain from the continuously compounded return series the following estimates for the standard deviations:

Period	IBM (weekly)	IBM (annual)	GE (weekly)	GE (annual)	XOM (weekly)	XOM (annual)
97/6 – 02/05	0.050	0.360	0.046	0.332	0.034	0.248
97/6 – 98/5	0.040	0.287	0.030	0.216	0.034	0.246
98/6 – 99/5	0.049	0.355	0.045	0.327	0.037	0.264
99/6 – 00/5	0.052	0.376	0.049	0.352	0.036	0.257
00/6 – 01/5	0.063	0.457	0.044	0.318	0.030	0.218
01/6 – 02/5	0.040	0.287	0.058	0.415	0.036	0.257

It seems that for IBM and GE, volatility increased over time, and slightly decreased towards the end of our sample period. The volatility of Exxon is relatively stable, with the year 00/01 having a particularly low volatility. The volatilities do not necessarily move in tandem. For example, the volatility of Exxon is lowest when both IBM and GE incur sharp increases in their volatilities.

Question 11.20.

We will use the methodology introduced by Hull, which is described in the main textbook. We can calculate:

$$\begin{array}{lll}
 u = 1.2005 & S = 50.0000 & K = 45.00 \\
 d = 0.8670 & F = 46.0792 & \text{dividend} = 4.0000 \\
 p = 0.4594 & t = 1.0000 & r = 0.0800 \\
 n = 4.0000 & h = 0.2500 & \text{sigma} = 0.3255 \\
 & & \text{time to div } 0.2500
 \end{array}$$

American Call			95.7188
			50.7188
		79.7304	
		35.6215	
	66.4127		69.1230
	23.1772		24.1230
	59.3195	57.5771	
	14.3195	13.4682	
50.0000	47.9597		49.9170
8.4551	7.2380		4.9170
43.3480		41.5791	
3.7876		2.2141	
	34.6340		36.0474
	0.9970		0.0000
		30.0263	
		0.0000	
			26.0315
			0.0000

Part 3 Options

European Call			95.7188
			50.7188
		79.7304	
		35.6215	
	66.4127		69.1230
	23.1772		24.1230
	59.3195	57.5771	
	14.2721	13.4682	
50.0000	47.9597		49.9170
8.4338	7.2380		4.9170
	43.3480	41.5791	
	3.7876	2.2141	
	34.6340		36.0474
	0.9970		0.0000
		30.0263	
		0.0000	
			26.0315
			0.0000