

## Chapter 3

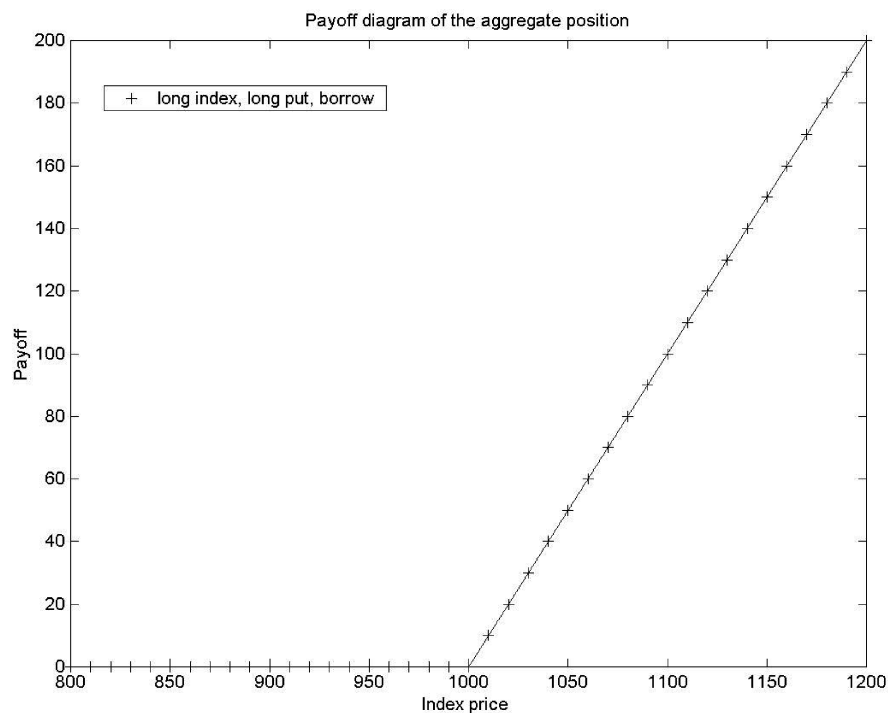
### Insurance, Collars, and Other Strategies

#### Question 3.1.

This question is a direct application of the Put-Call-Parity (equation (3.1)) of the textbook. Mimicking Table 3.1., we have:

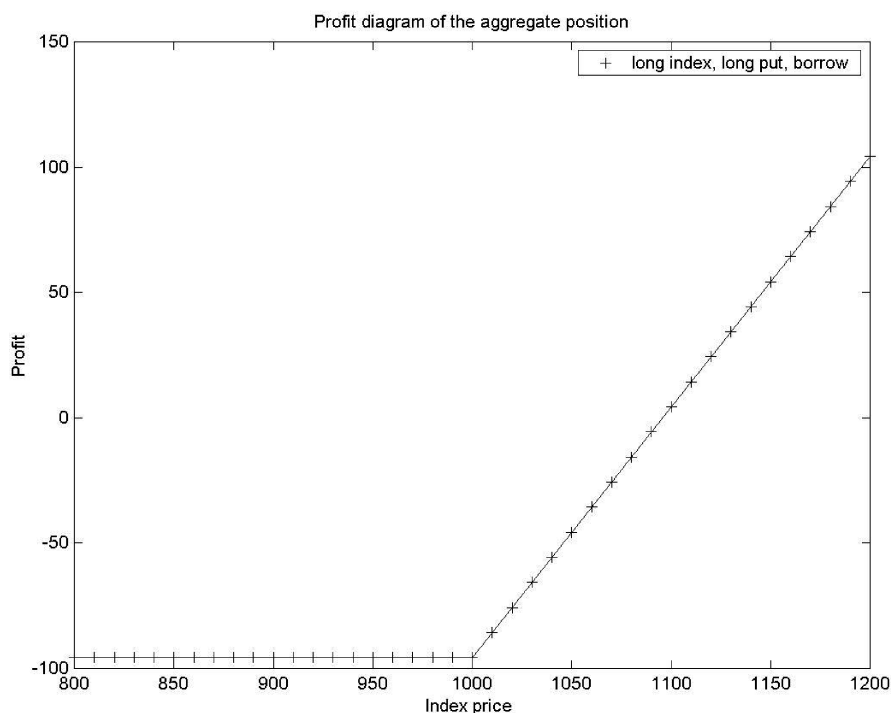
S&R Index	S&R Put	Loan	Payoff	$-(\text{Cost} + \text{Interest})$	Profit
900.00	100.00	-1000.00	0.00	-95.68	-95.68
950.00	50.00	-1000.00	0.00	-95.68	-95.68
1000.00	0.00	-1000.00	0.00	-95.68	-95.68
1050.00	0.00	-1000.00	50.00	-95.68	-45.68
1100.00	0.00	-1000.00	100.00	-95.68	4.32
1150.00	0.00	-1000.00	150.00	-95.68	54.32
1200.00	0.00	-1000.00	200.00	-95.68	104.32

The payoff diagram looks as follows:



We can see from the table and from the payoff diagram that we have in fact reproduced a call with the instruments given in the exercise. The profit diagram on the next page confirms this hypothesis.





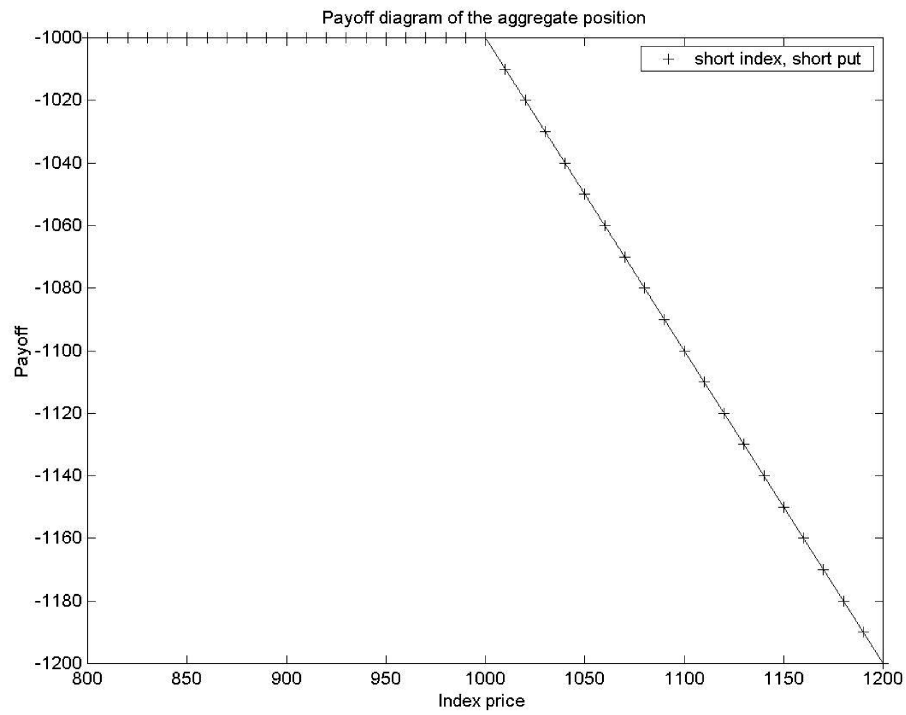
### Question 3.2.

This question constructs a position that is the opposite to the position of Table 3.1. Therefore, we should get the exact opposite results from Table 3.1. and the associated figures. Mimicking Table 3.1., we indeed have:

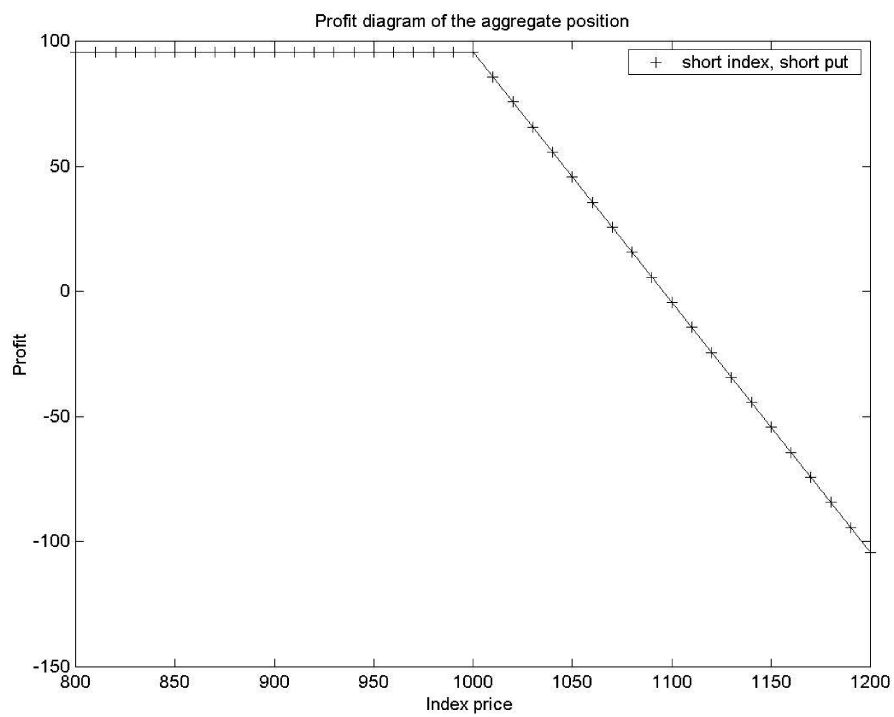
S&R Index	S&R Put	Payoff	-(Cost + Interest)	Profit
-900.00	-100.00	-1000.00	1095.68	95.68
-950.00	-50.00	-1000.00	1095.68	95.68
-1000.00	0.00	-1000.00	1095.68	95.68
-1050.00	0.00	-1050.00	1095.68	45.68
-1100.00	0.00	-1100.00	1095.68	-4.32
-1150.00	0.00	-1150.00	1095.68	-54.32
-1200.00	0.00	-1200.00	1095.68	-104.32

On the next page, we see the corresponding payoff and profit diagrams. Please note that they match the combined payoff and profit diagrams of Figure 3.5. Only the axes have different scales.

Payoff-diagram:



Profit diagram:



### Question 3.3.

In order to be able to draw profit diagrams, we need to find the future value of the put premium, the call premium and the investment in zero-coupon bonds. We have for:

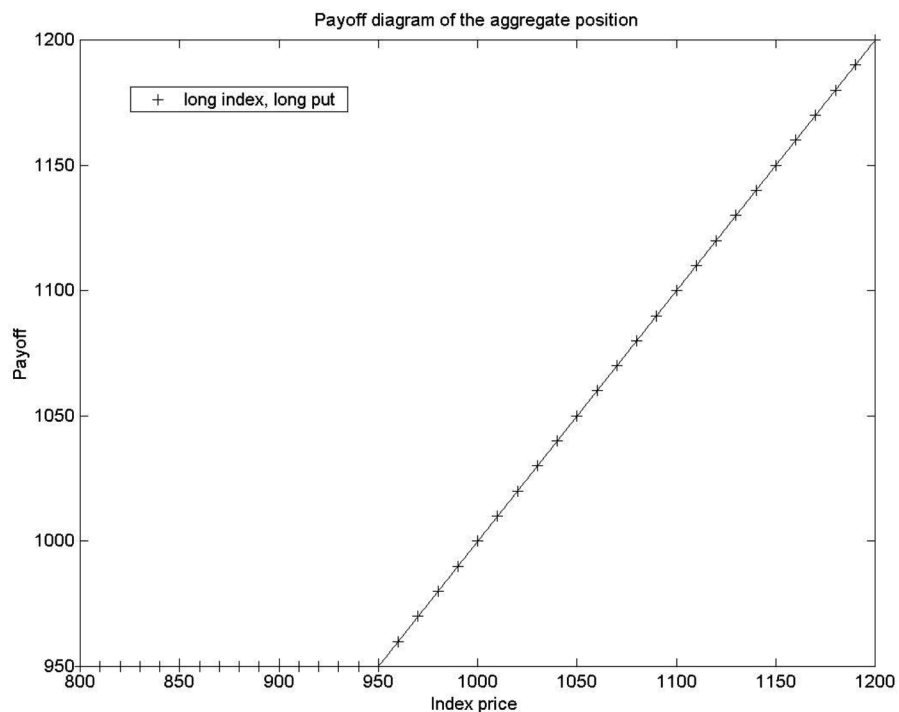
the put premium:  $\$51.777 \times (1 + 0.02) = \$52.81$ ,

the call premium:  $\$120.405 \times (1 + 0.02) = \$122.81$  and

the zero-coupon bond:  $\$931.37 \times (1 + 0.02) = \$950.00$

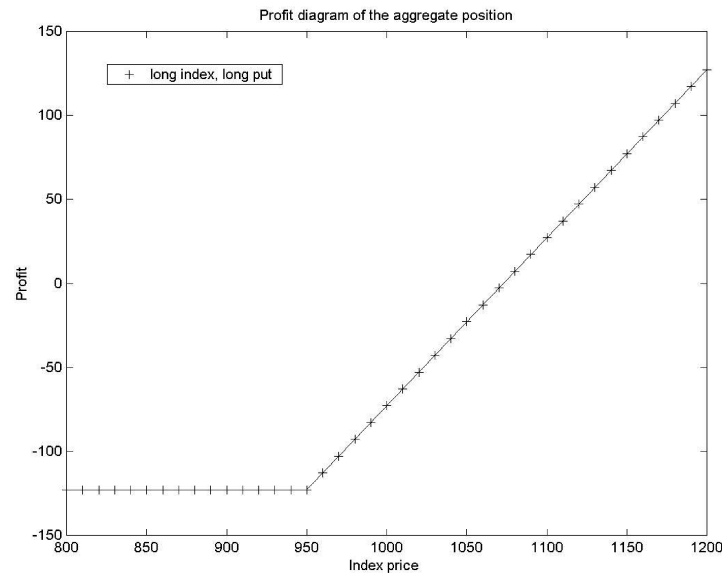
Now, we can construct the payoff and profit diagrams of the aggregate position:

Payoff diagram:



From this figure, we can already see that the combination of a long put and the long index looks exactly like a certain payoff of \$950, plus a call with a strike price of 950. But this is the alternative given to us in the question. We have thus confirmed the equivalence of the two combined positions for the payoff diagrams. The profit diagrams on the next page confirm the equivalence of the two positions (which is again an application of the Put-Call-Parity).

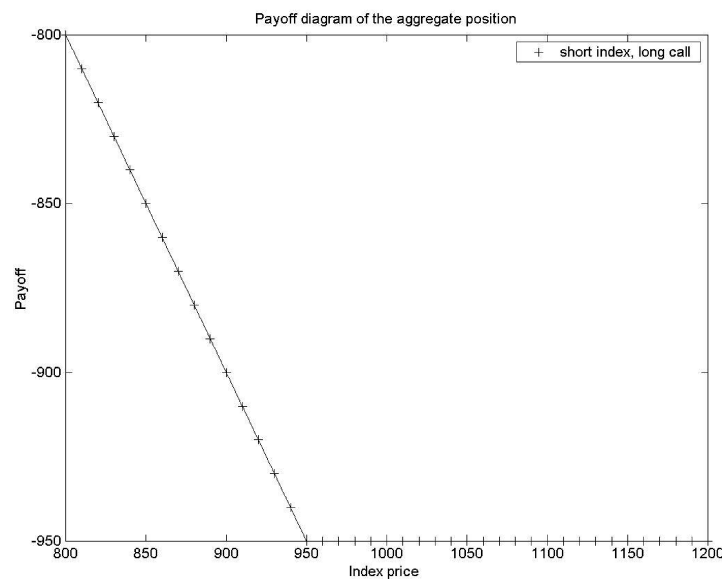
Profit Diagram for a long 950-strike put and a long index combined:



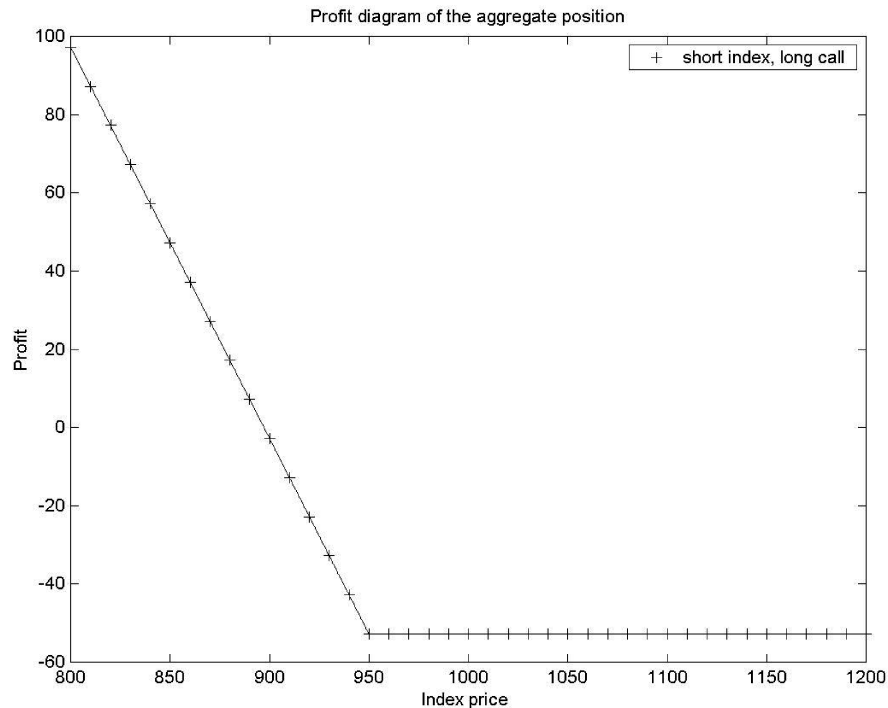
#### Question 3.4.

This question is another application of Put-Call-Parity. Initially, we have the following cost to enter into the combined position: We receive \$1,000 from the short sale of the index, and we have to pay the call premium. Therefore, the future value of our cost is:  $(\$120.405 - \$1,000) \times (1 + 0.02) = -\$897.19$ . Note that a negative cost means that we initially have an inflow of money.

Now, we can directly proceed to draw the payoff diagram:



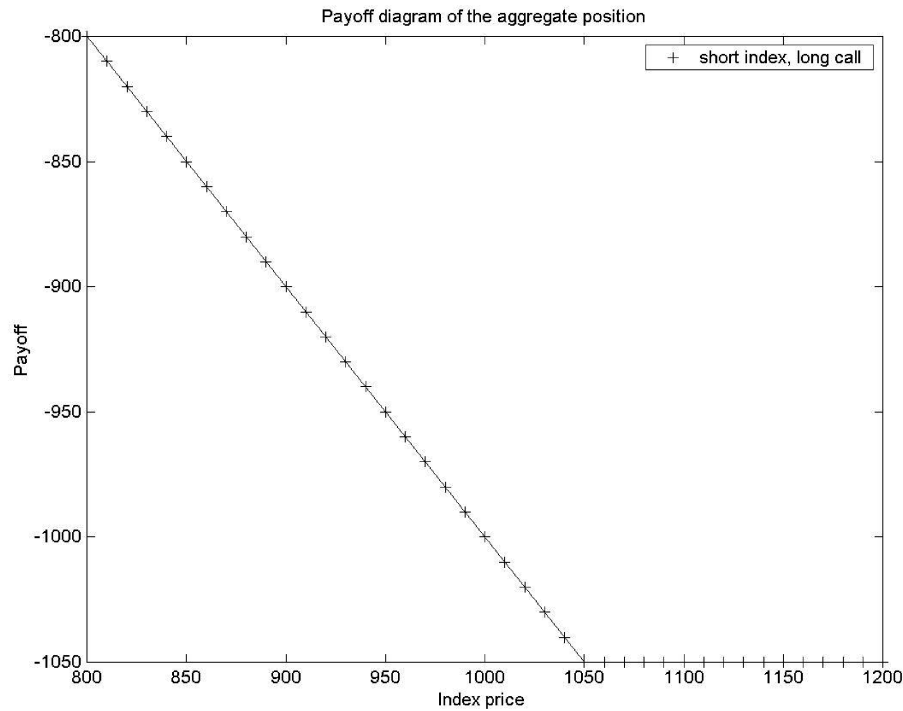
We can clearly see from the figure that the payoff graph of the short index and the long call looks like a fixed obligation of \$950, which is alleviated by a long put position with a strike price of 950. The following profit diagram, including the cost for the combined position, confirms this:



### Question 3.5.

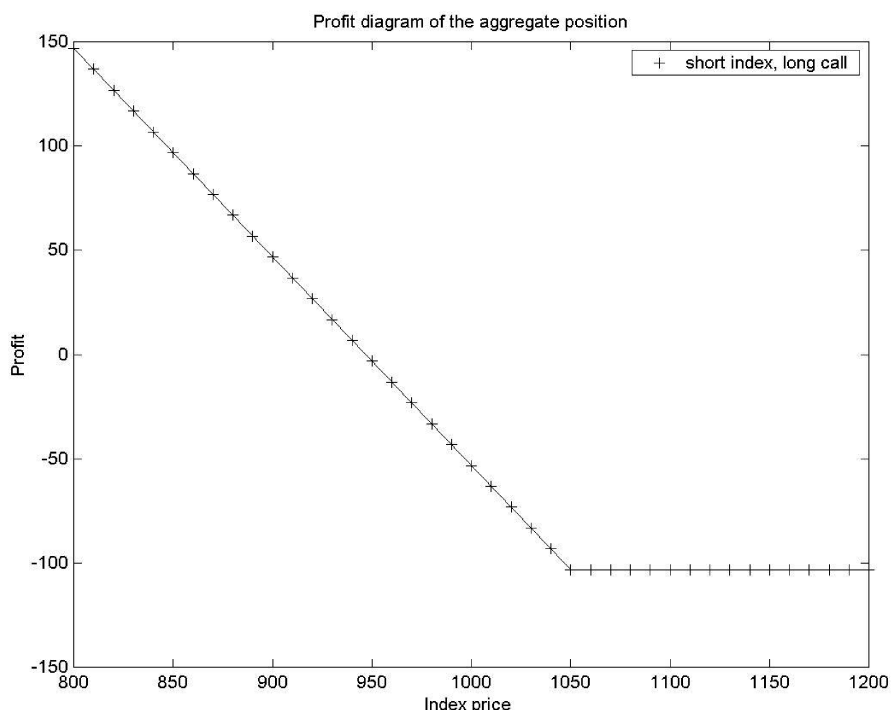
This question is another application of Put-Call-Parity. Initially, we have the following cost to enter into the combined position: We receive \$1,000 from the short sale of the index, and we have to pay the call premium. Therefore, the future value of our cost is:  $(\$71.802 - \$1,000) \times (1 + 0.02) = -\$946.76$ . Note that a negative cost means that we initially have an inflow of money.

Now, we can directly proceed to draw the payoff diagram:



In order to be able to compare this position to the other suggested position of the exercise, we need to find the future value of the borrowed \$1,029.41. We have:  $\$1,029.41 \times (1 + 0.02) = \$1,050$ . We can now see from the figure that the payoff graph of the short index and the long call looks like a fixed obligation of \$1,050, which is exactly the future value of the borrowed amount, and a long put position with a strike price of 1,050. The following profit diagram, including the cost for the combined position we calculated above, confirms this. The profit diagram is the same as the profit diagram for a loan and a long 1,050-strike put with an initial premium of \$101.214.

Profit Diagram of going short the index and buying a 1,050-strike call:



### Question 3.6.

We now move from a graphical representation and verification of the Put-Call-Parity to a mathematical representation. Let us first consider the payoff of (a). If we buy the index (let us name it  $S$ ), we receive at the time of expiration  $T$  of the options simply  $S_T$ .

The payoffs of part (b) are a little bit more complicated. If we deal with options and the maximum function, it is convenient to split the future values of the index into two regions: one where  $S_T < K$  and another one where  $S_T \geq K$ . We then look at each region separately, and hope to be able to draw a conclusion when we look at the aggregate position.

We have for the payoffs in (b):

Instrument	$S_T < K = 950$	$S_T \geq K = 950$
Get repayment of loan	$\$931.37 \times 1.02 = \$950$	$\$931.37 \times 1.02 = \$950$
Long Call Option	$\max(S_T - 950, 0) = 0$	$S_T - 950$
Short Put Option	$-\max(950 - S_T, 0)$ $= S_T - 950$	0
Total	$S_T$	$S_T$

We now see that the total aggregate position only gives us  $S_T$ , no matter what the final index value is—but this is the same payoff as in part (a). Our proof for the payoff equivalence is complete.

Now let us turn to the profits. If we buy the index today, we need to finance it. Therefore, we borrow \$1,000, and have to repay \$1,020 after one year. The profit for part (a) is thus:  $S_T - \$1,020$ .

The profits of the aggregate position in part (b) are the payoffs, less the future value of the call premium plus the future value of the put premium (because we have sold the put), and less the future value of the loan we gave initially. Note that we already know that a risk-less bond is canceling out of the profit calculations. We can again tabulate:

Instrument	$S_T < K$	$S_T \geq K$
Get repayment of loan	$\$931.37 \times 1.02 = \$950$	$\$931.37 \times 1.02 = \$950$
Future value of given loan	$-\$950$	$-\$950$
Long Call Option	$\max(S_T - 950, 0) = 0$	$S_T - 950$
Future value call premium	$-\$120.405 \times 1.02 = -\$122.81$	$-\$120.405 \times 1.02 = -\$122.81$
Short Put Option	$-\max(\$950 - S_T, 0)$ $= S_T - \$950$	$0$
Future value put premium	$\$51.777 \times 1.02 = \$52.81$	$\$51.777 \times 1.02 = \$52.81$
Total	$S_T - 1020$	$S_T - 1020$

Indeed, we see that the profits for part (a) and part (b) are identical as well.

### Question 3.7.

Let us first consider the payoff of (a). If we short the index (let us name it  $S$ ), we have to pay at the time of expiration  $T$  of the options:  $-S_T$ .

The payoffs of part (b) are more complicated. Let us look again at each region separately, and hope to be able to draw a conclusion when we look at the aggregate position.

We have for the payoffs in (b):

Instrument	$S_T < K$	$S_T \geq K$
Make repayment of loan	$-\$1029.41 \times 1.02 = -\$1050$	$-\$1029.41 \times 1.02 = -\$1050$
Short Call Option	$-\max(S_T - 1050, 0) = 0$	$-\max(S_T - 1050, 0)$ $= 1050 - S_T$
Long Put Option	$\max(\$1050 - S_T, 0)$ $= \$1050 - S_T$	$0$
Total	$-S_T$	$-S_T$



We see that the total aggregate position gives us  $-S_T$ , no matter what the final index value is—but this is the same payoff as part (a). Our proof for the payoff equivalence is complete.

Now let us turn to the profits. If we sell the index today, we receive money that we can lend out. Therefore, we can lend \$1,000, and be repaid \$1,020 after one year. The profit for part (a) is thus:  $\$1,020 - S_T$ .

The profits of the aggregate position in part (b) are the payoffs, less the future value of the put premium plus the future value of the call premium (because we sold the call), and less the future value of the loan we gave initially. Note that we know already that a risk-less bond is canceling out of the profit calculations. We can again tabulate:

Instrument	$S_T < K$	$S_T \geq K$
Make repayment of loan	$-\$1029.41 \times 1.02 = -\$1050$	$-\$1029.41 \times 1.02 = -\$1050$
Future value of borrowed money	\$1050	\$1050
Short Call Option	$-\max(S_T - 1050, 0) = 0$	$-\max(S_T - 1050, 0) = 1050 - S_T$
Future value of premium	$\$71.802 \times 1.02 = \$73.24$	$\$71.802 \times 1.02 = \$73.24$
Long Put Option	$\max(\$1050 - S_T, 0) = \$1050 - S_T$	0
Future value of premium	$-\$101.214 \times 1.02 = -\$103.24$	$-\$101.214 \times 1.02 = -\$103.24$
Total	$\$1,020 - S_T$	$\$1,020 - S_T$

Indeed, we see that the profits for part (a) and part (b) are identical as well.

### Question 3.8.

This question is a direct application of the Put-Call-Parity. We will use equation (3.1) in the following, and input the given variables:

$$\begin{aligned}
 & Call(K, t) - Put(K, t) = PV(F_{0,t} - K) \\
 \Leftrightarrow & Call(K, t) - Put(K, t) - PV(F_{0,t}) = -PV(K) \\
 \Leftrightarrow & Call(K, t) - Put(K, t) - S_0 = -PV(K) \\
 \Leftrightarrow & \$109.20 - \$60.18 - \$1,000 = -\frac{K}{1.02} \\
 \Leftrightarrow & K = \$970.00
 \end{aligned}$$

### Question 3.9.

The strategy of buying a call (or put) and selling a call (or put) at a higher strike is called call (put) bull spread. In order to draw the profit diagrams, we need to find the future value of the cost of entering in the bull spread positions. We have:

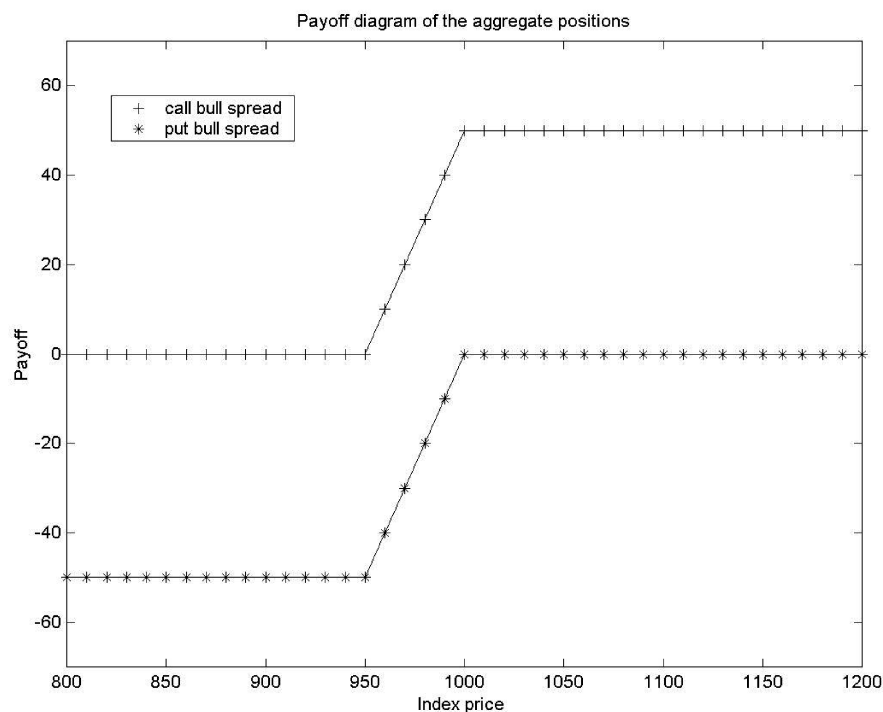
Cost of call bull spread:  $(\$120.405 - \$93.809) \times 1.02 = \$27.13$

Cost of put bull spread:  $(\$51.777 - \$74.201) \times 1.02 = -\$22.87$

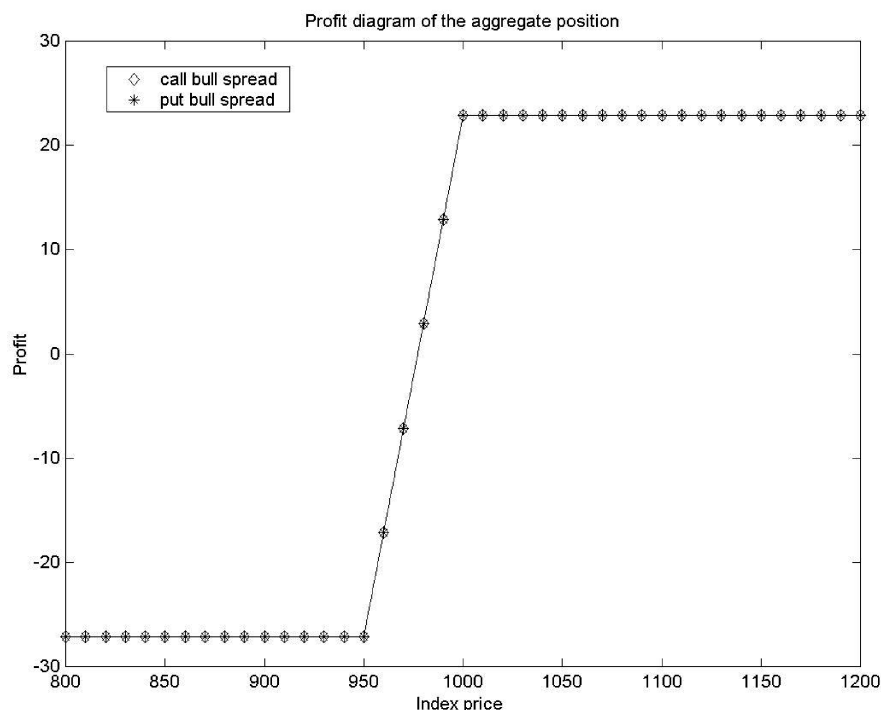
The payoff diagram shows that the payoffs to the put bull spread are uniformly less than the payoffs to the call bull spread. There is a difference, because the put bull spread has a negative initial cost, i.e., we are receiving money if we enter into it. The difference is exactly \$50, the value of the difference between the two strike prices. It is equivalent as well to the value of the difference of the future values of the initial premia.

Yet, the higher initial cost for the call bull spread is exactly offset by the higher payoff so that the profits of both strategies are the same. It is easy to show this with equation (3.1), the put-call-parity.

Payoff-Diagram:



Profit diagram:



### Question 3.10.

The strategy of selling a call (or put) and buying a call (or put) at a higher strike is called call (put) bear spread. In order to draw the profit diagrams, we need to find the future value of the cost of entering in the bull spread positions. We have:

Cost of call bear spread:  $(\$71.802 - \$120.405) \times 1.02 = -\$49.575$

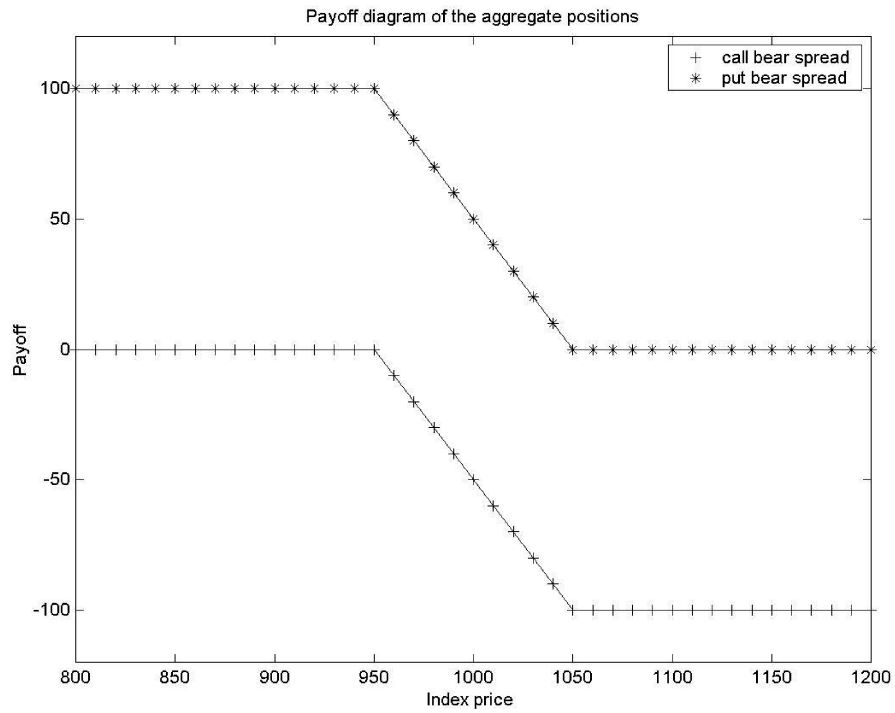
Cost of put bear spread:  $(\$101.214 - \$51.777) \times 1.02 = \$50.426$

The payoff diagram shows that the payoff to the call bear spread is uniformly less than the payoffs to the put bear spread. The difference is exactly \$100, equal to the difference in strikes and as well equal to the difference in the future value of the costs of the spreads.

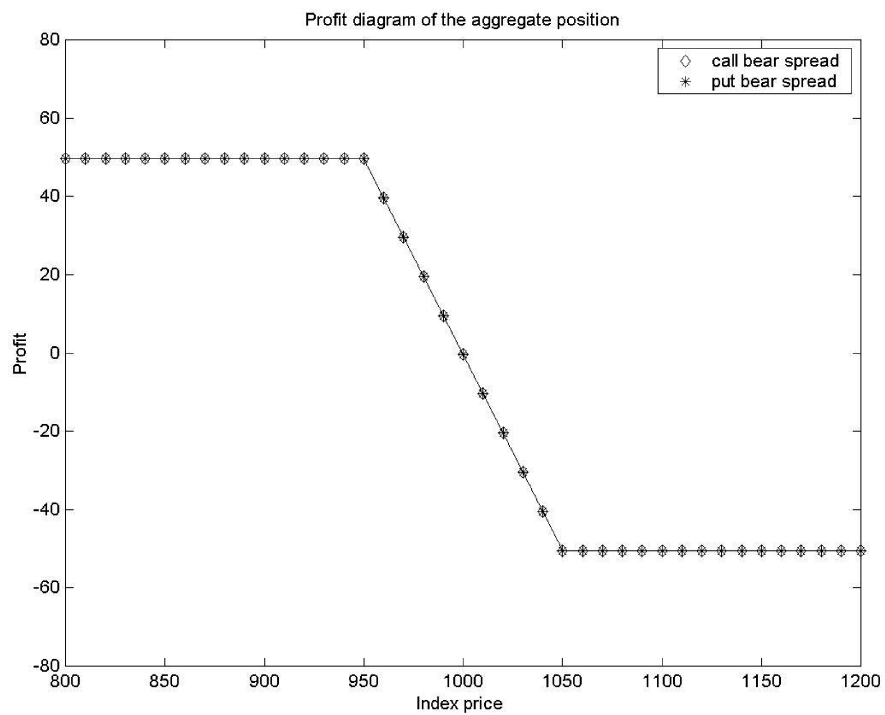
There is a difference, because the call bear spread has a negative initial cost, i.e., we are receiving money if we enter into it.

The higher initial cost for the put bear spread is exactly offset by the higher payoff so that the profits of both strategies turn out to be the same. It is easy to show this with equation (3.1), the put-call-parity.

Payoff-Diagram:



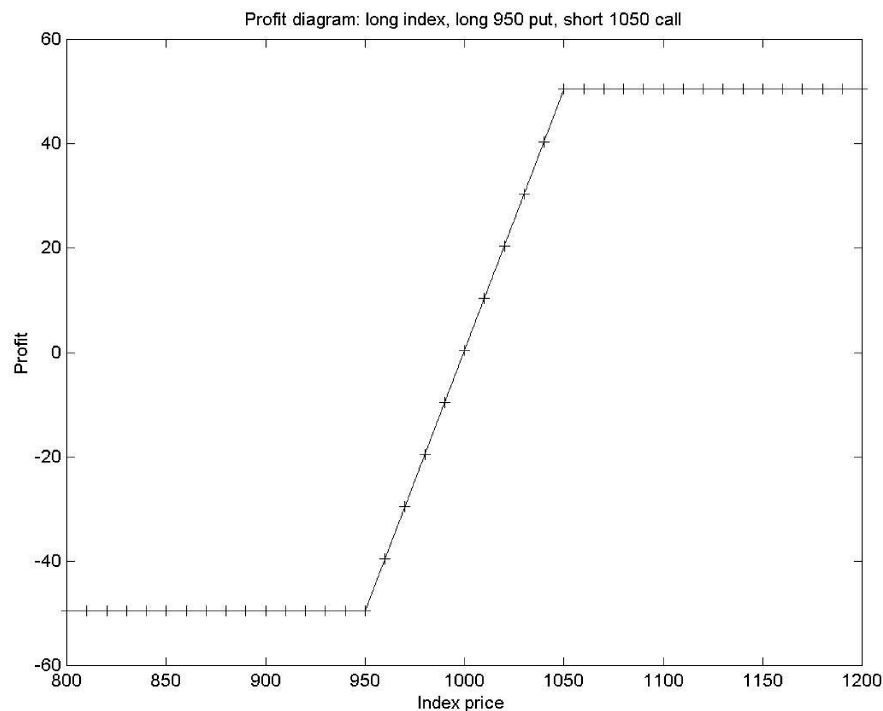
Profit Diagram:



### Question 3.11.

In order to be able to draw the profit diagram, we need to find the future value of the costs of establishing the suggested position. We need to finance the index purchase, buy the 950-strike put and we receive the premium of the sold call. Therefore, the future value of our cost is:  $(\$1,000 - \$71.802 + \$51.777) \times 1.02 = \$999.57$ .

Now we can draw the profit diagram:

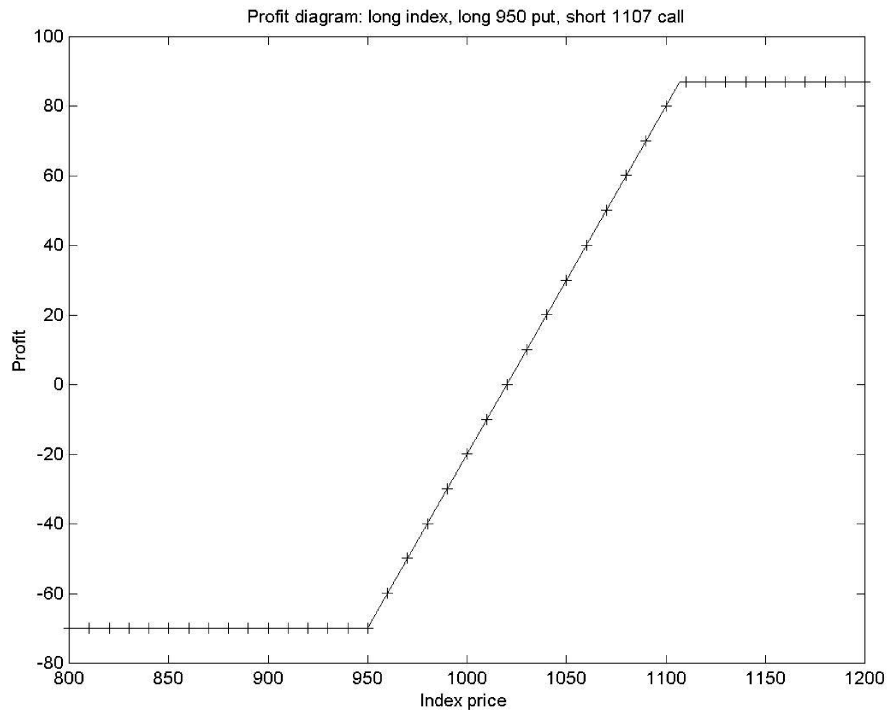


The net option premium cost today is:  $-\$71.802 + \$51.777 = -\$20.025$ . We receive about \$20 if we enter into this collar. If we want to construct a zero-cost collar and keep the 950-strike put, we would need to increase the strike price of the call. By increasing the strike price of the call, the buyer of the call must wait for larger increases in the underlying index before the option pays off. This makes the call option less attractive, and the buyer of the option is only willing to pay a smaller premium. We receive less money, thus pushing the net option premium towards zero.

### Question 3.12.

Our initial cash required to put on the collar, i.e. the net option premium, is as follows:  $-\$51.873 + \$51.777 = -\$0.096$ . Therefore, we receive only 10 cents if we enter into this collar. The position is very close to a zero-cost collar.

The profit diagram looks as follows:

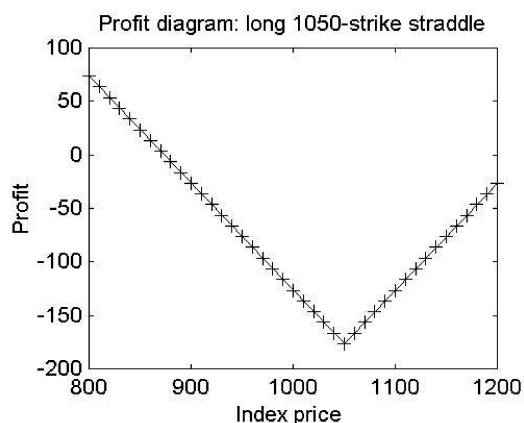


If we compare this profit diagram with the profit diagram of the previous exercise (3.11.), we see that we traded in the additional call premium (that limited our losses after index decreases) against more participation on the upside. Please note that both maximum loss and gain are higher than in question 3.11.

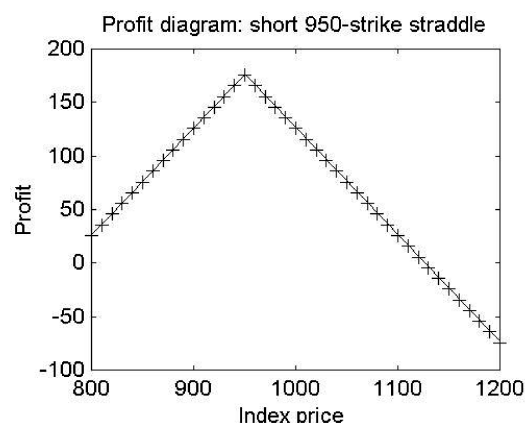
### Question 3.13.

The following figure depicts the requested profit diagrams. We can see that the aggregation of the bought and sold straddle resembles a bear spread. It is bearish, because we sold the straddle with the smaller strike price.

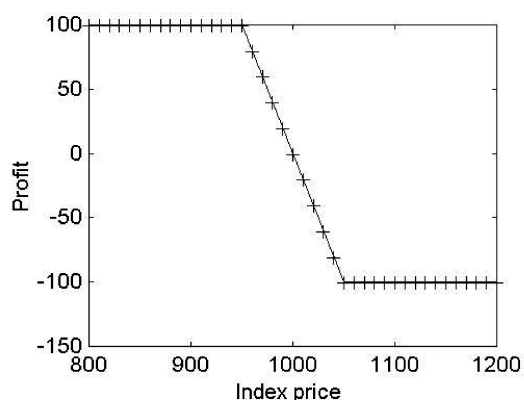
a)



b)



c)



Profit diagram: long 1050-strike,  
short 950-strike straddle

### Question 3.14.

a) This question deals with the option trading strategy known as Box spread. We saw earlier that if we deal with options and the maximum function, it is convenient to split the future values of the index into different regions. Let us name the final value of the S&R index  $S_T$ . We have two strike prices, therefore we will use three regions: One in which  $S_T < 950$ , one in which  $950 \leq S_T < 1,000$  and another one in which  $S_T \geq 1,000$ . We then look at each region separately, and hope to be able to see that indeed when we aggregate, there is no S&R risk when we look at the aggregate position.

Instrument	$S_T < 950$	$950 \leq S_T < 1,000$	$S_T \geq 1,000$
long 950 call	0	$S_T - \$950$	$S_T - \$950$
short 1000 call	0	0	$\$1,000 - S_T$
short 950 put	$S_T - \$950$	0	0
long 1000 put	$\$1,000 - S_T$	$\$1,000 - S_T$	0
Total	\$50	\$50	\$50

We see that there is no occurrence of the final index value in the row labeled total. The option position does not contain S&R price risk.

b) The initial cost is the sum of the long option premia less the premia we receive for the sold options. We have:

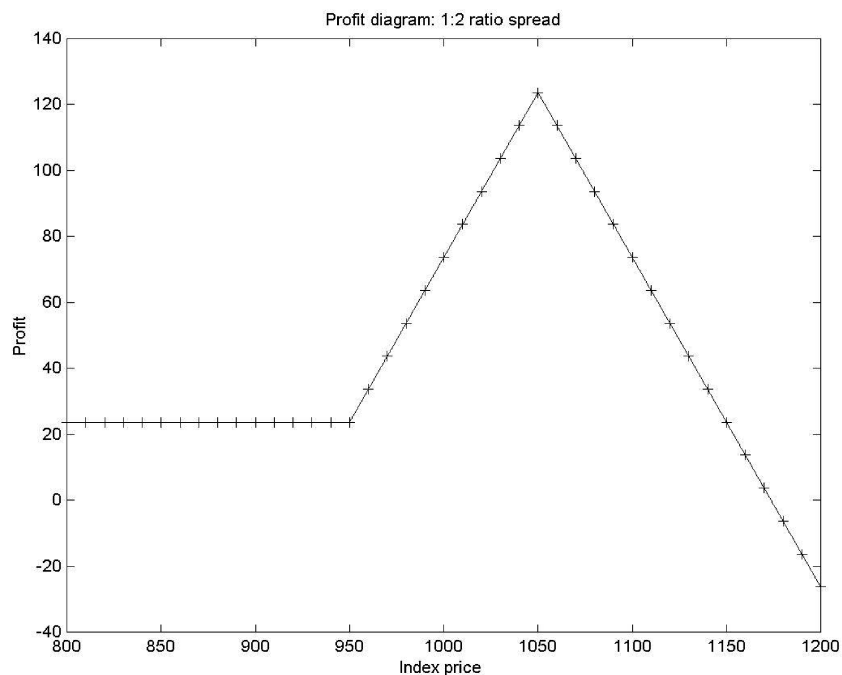
$$\text{Cost } \$120.405 - \$93.809 - \$51.77 + \$74.201 = \$49.027$$

c) The payoff of the position after 6 months is \$50, as we can see from the above table.

d) The implicit interest rate of the cash flows is:  $\$50.00 \div \$49.027 = 1.019 \cong 1.02$ . The implicit interest rate is indeed 2 percent.

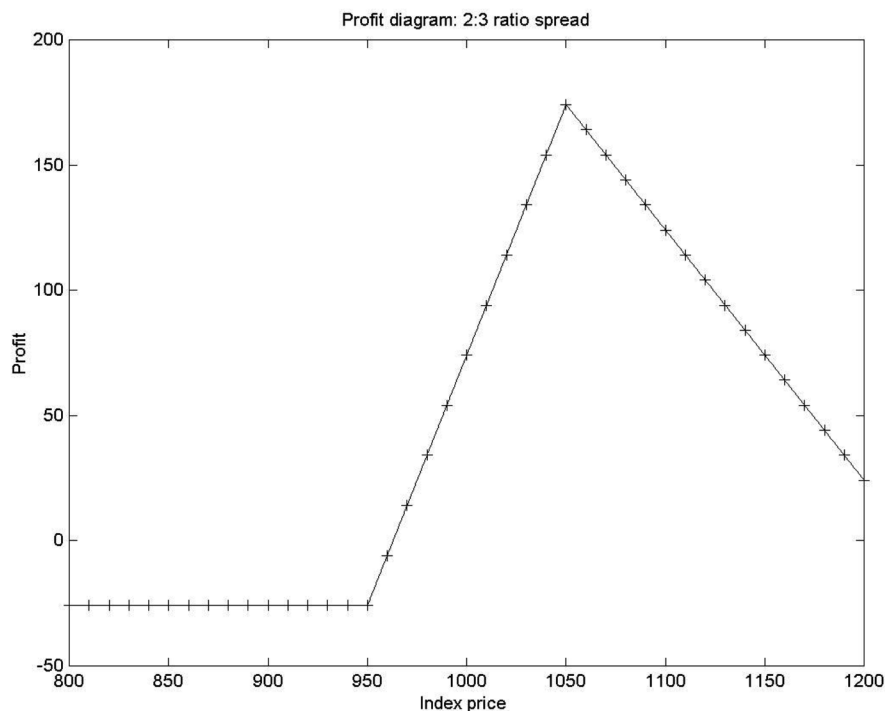
### Question 3.15.

a) Profit diagram of the 1:2 950-, 1050-strike ratio call spread (the future value of the initial cost of which is calculated as:  $(\$120.405 - 2 \times \$71.802) \times 1.02 = -\$23.66$ ):



b) Profit diagram of the 2:3 950-, 1050-strike ratio call spread (the future value of the initial cost of which is calculated as:  $(2 \times \$120.405 - 3 \times \$71.802) \times 1.02 = \$25.91$ ).





c) We saw that in part a), we were receiving money from our position, and in part b), we had to pay a net option premium to establish the position. This suggests that the true ratio  $n/m$  lies between 1:2 and 2:3.

Indeed, we can calculate the ratio  $n/m$  as:

$$\begin{aligned}
 n \times \$120.405 - m \times \$71.802 &= 0 \\
 \Leftrightarrow n \times \$120.405 &= m \times \$71.802 \\
 \Leftrightarrow n/m &= \$71.802/\$120.405 \\
 \Leftrightarrow n/m &= 0.596
 \end{aligned}$$

which is approximately 3:5.

### Question 3.16.

A bull spread or a bear spread can never have an initial premium of zero, because we are buying the same number of calls (or puts) that we are selling and the two legs of the bull and bear spreads have different strikes. A zero initial premium would mean that two calls (or puts) with different strikes have the same price—and we know by now that two instruments that have different payoff structures and the same underlying risk cannot have the same price without creating an arbitrage opportunity.

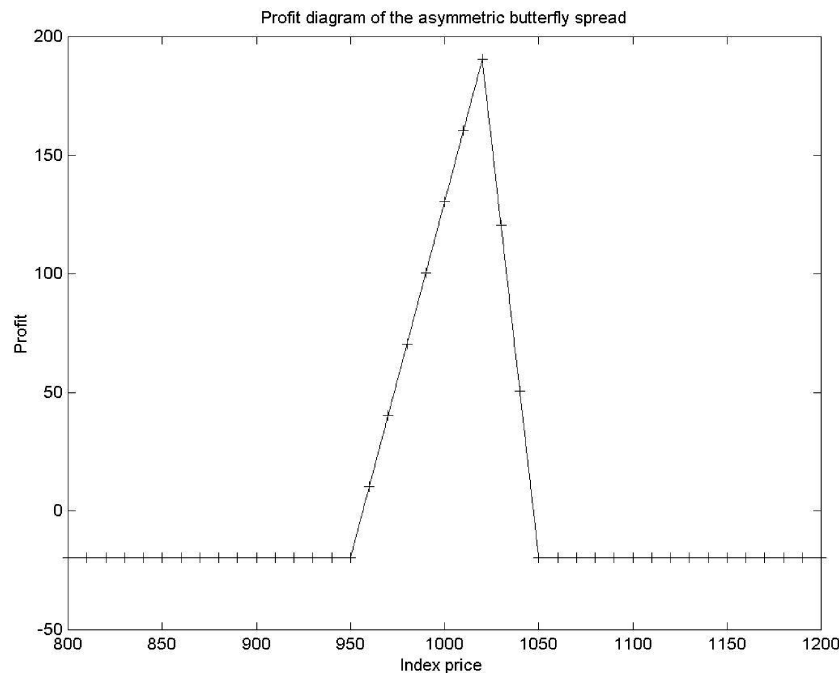
A symmetric butterfly spread cannot have a premium of zero because it would violate the convexity condition of options.

**Question 3.17.**

From the textbook we learn how to calculate the right ratio  $\lambda$ . It is equal to:

$$\lambda = \frac{K_3 - K_2}{K_3 - K_1} = \frac{1050 - 1020}{1050 - 950} = 0.3$$

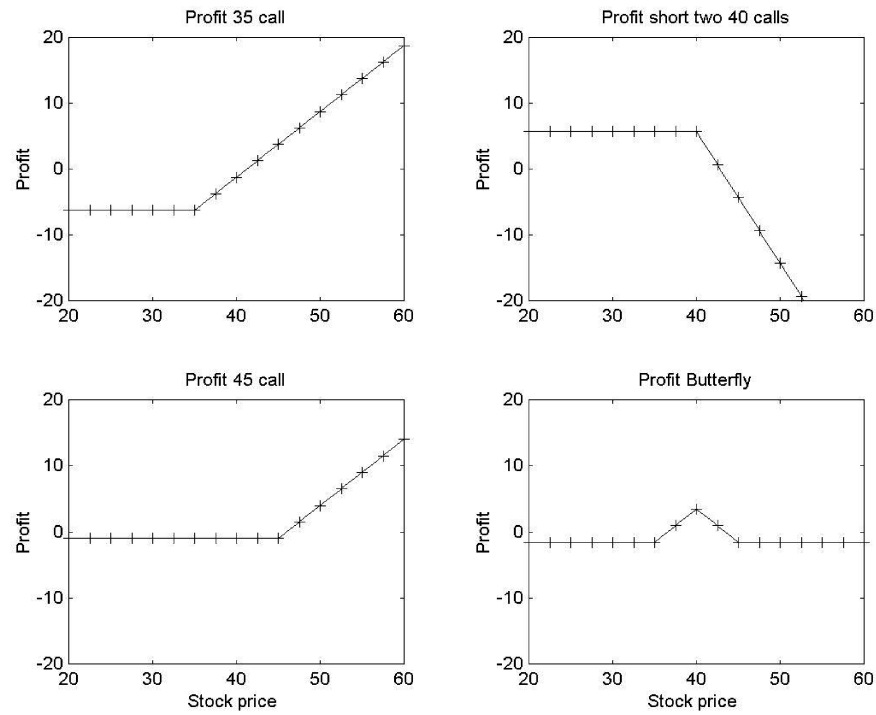
In order to construct the asymmetric butterfly, for every 1020-strike call we write, we buy  $\lambda$  950-strike calls and  $1 - \lambda$  1050-strike calls. Since we can only buy whole units of calls, we will in this example buy three 950-strike and seven 1050-strike calls, and sell ten 1020-strike calls. The profit diagram looks as follows:

**Question 3.18.**

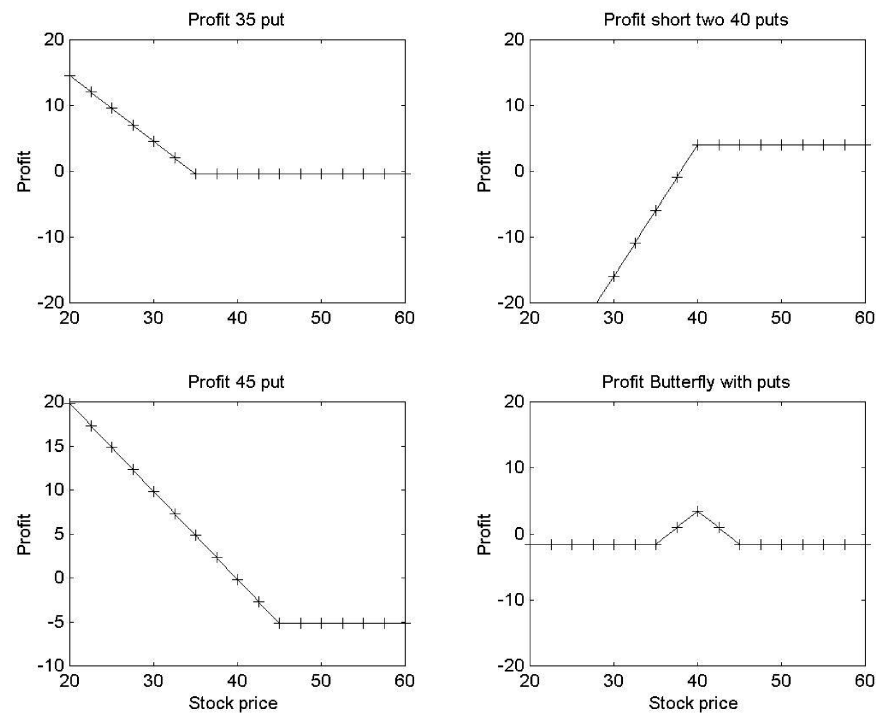
The following three figures show the individual legs of each of the three suggested strategies. The last subplot shows the aggregate position. It is evident from the figures that you can indeed use all the suggested strategies to construct the same butterfly spread. Another method to show the claim of 3.18. mathematically would be to establish the equivalence by using the Put-Call-Parity on b) and c) and showing that you can write it in terms of the instruments of a).

## Part 1 Insurance, Hedging, and Simple Strategies

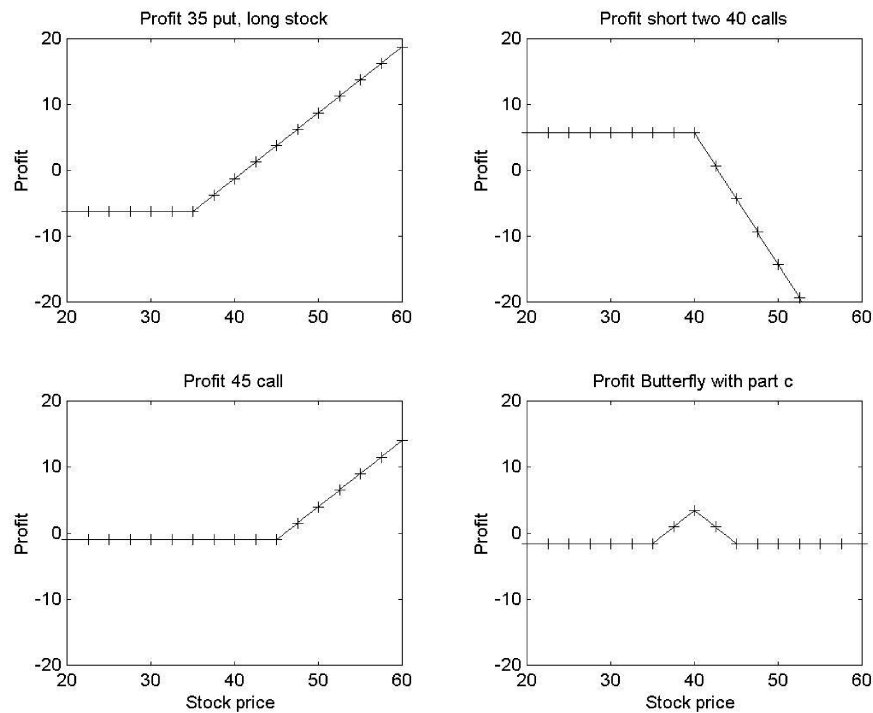
profit diagram part a)



profit diagram part b)



profit diagram part c)



### Question 3.19.

- a) We know from the Put-Call-Parity that if we buy a call and sell a put that are at the money (i.e.,  $S(0) = K$ ), then the call option is slightly more expensive than the put option, the difference being the value of the stock minus the present value of the strike. Therefore, we can tell that the strike price must be a bit higher than the current stock price, and more precisely, it should be equal to the forward price.
- b) We sold a collar with no difference in strike prices. The profit diagram will be a straight line, which means that we effectively created a long forward contract.
- c) Remember that you are buying at the ask and selling at the bid, and that the bid price is always smaller than the ask. Suppose we had established a zero-cost synthetic at the forward price, and now we introduce the bid-ask spread. This means that we have to pay a little more for the call, and receive a little less for the put. We are paying money for the position, and in order to correct it, we must make the put a bit more attractive, and the call less attractive. We do so by shifting the strike price to the right of the forward: Now the buyer of the call must wait a little bit longer before his call pays off, and he is only willing to buy it for less. As the opposite is true for the put, we have established that the strike must be to the right of the forward.

d) If we are creating a synthetic short stock, we buy the put option and sell the call option. We are buying at the ask and selling at the bid, and the bid price is always smaller than the ask. Suppose we had established a zero-cost synthetic short at the forward price, and now we introduce the bid-ask spread. This means that we have to pay a little more for the put, and receive a little less for the call. We are paying money for the position, and in order to correct it, we must make the call a bit more attractive. The call gets more attractive if the strike price decreases, because the final payoff is  $\max(S - K, 0)$ . Therefore, we have to shift the strike price to the left of the forward price.

e) No, transaction fees are not a wash, because we are paying implicitly the bid-ask spread: If we bought a stock today and held it until the expiration of the options, we would get the future stock price less the forward price (which is equivalent to the loan we got to finance the stock purchase). Now, we established in c) that the strike price is to the right of the forward price. Therefore, we will receive from the collar of part c) the stock price less something that is larger than the forward price: We make a loss compared to the self-financed outright purchase of the stock. These considerations do not yet take into account that we incur transaction costs on two instruments, compared to only one time brokerage fees if we buy the stock directly.

It is thus very important to be aware of transaction costs when comparing different investment strategies.

### Question 3.20.

Use separate cells for the strike price and the quantities you buy and sell for each strike (i.e., make use of the plus or minus sign). Then, use the maximum function to calculate payoffs and profits.

The best way to solve this problem is probably to have the calculations necessary for the payoff and profit diagrams run in the background, e.g., in another auxiliary table that you are referencing to. Define the boundaries for the calculations dynamically and symmetrically around the current stock price. Then use the diagram function with the line style to draw the diagrams.