Chapter 5
Financial Forwards and Futures

Question 5.1.

Four different ways to sell a share of stock that has a price S(0) at time 0.

Description	Get Paid at Time	Lose Ownership of Security at Time	Receive Payment of
Outright Sale	0	0	S_0 at time o
Security Sale and Loan Sale	T	0	$S_0 e^{rT}$ at time T
Short Prepaid Forward Contract	0	T	?
Short Forward Contract	T	T	? $\times e^{rT}$

Question 5.2.

a) The owner of the stock is entitled to receive dividends. As we will get the stock only in one year, the value of the prepaid forward contract is today's stock price, less the present value of the four dividend payments:

$$F_{0,T}^P = \$50 - \sum_{i=1}^{4} \$1 e^{-0.06\times\frac{3}{12}i} = \$50 - \$0.985 - \$0.970 - \$0.956 - \$0.942$$
$$= \$50 - \$3.853 = \$46.147$$

b) The forward price is equivalent to the future value of the prepaid forward. With an interest rate of 6 percent and an expiration of the forward in one year, we thus have:

$$F_{0,T} = F_{0,T}^P \times e^{0.06\times 1} = \$46.147 \times e^{0.06\times 1} = \$46.147 \times 1.0618 = \$49.00$$

Question 5.3.

a) The owner of the stock is entitled to receive dividends. We have to offset the effect of the continuous income stream in form of the dividend yield by tailing the position:

$$F_{0,T}^P = \$50 e^{-0.08\times 1} = \$50 \times 0.9231 = \$46.1558$$

We see that the value is very similar to the value of the prepaid forward contract with discrete dividends we have calculated in question 5.2. In question 5.2., we received four cash dividends, with payments spread out through the entire year, totaling \$4. This yields a total annual dividend yield of approximately $\$4 \div \$50 = 0.08$.

b) The forward price is equivalent to the future value of the prepaid forward. With an interest rate of 6 percent and an expiration of the forward in one year we thus have:

$$F_{0,T} = F_{0,T}^{P} \times e^{0.06\times 1} = \$46.1558 \times e^{0.06\times 1} = \$46.1558 \times 1.0618 = \$49.01$$

Question 5.4.

This question asks us to familiarize ourselves with the forward valuation equation.

a) We plug the continuously compounded interest rate and the time to expiration in years into the valuation formula and notice that the time to expiration is 6 months, or 0.5 years. We have:

$$F_{0,T} = S_0 \times e^{r\times T} = \$35 \times e^{0.05\times 0.5} = \$35 \times 1.0253 = \$35.886$$

b) The annualized forward premium is calculated as:

$$\text{annualized forward premium} = \frac{1}{T}\ln\left(\frac{F_{0,T}}{S_0}\right) = \frac{1}{0.5}\ln\left(\frac{\$35.50}{\$35}\right) = 0.0284$$

c) For the case of continuous dividends, the forward premium is simply the difference between the risk-free rate and the dividend yield:

$$\begin{aligned}\text{annualized forward premium} &= \frac{1}{T}\ln\left(\frac{F_{0,T}}{S_0}\right) = \frac{1}{T}\ln\left(\frac{S_0 \times e^{(r-\delta)T}}{S_0}\right)\\ &= \frac{1}{T}\ln\left(e^{(r-\delta)T}\right) = \frac{1}{T}(r-\delta)\,T\\ &= r-\delta\end{aligned}$$

Therefore, we can solve:

$$\begin{aligned} & 0.0284 = 0.05 - \delta \\ \Leftrightarrow\ & \delta \quad\quad = 0.0216\end{aligned}$$

The annualized dividend yield is 2.16 percent.

Question 5.5.

a) We plug the continuously compounded interest rate and the time to expiration in years into the valuation formula and notice that the time to expiration is 9 months, or 0.75 years. We have:

$$F_{0,T} = S_0 \times e^{r\times T} = \$1{,}100 \times e^{0.05\times 0.75} = \$1{,}100 \times 1.0382 = \$1{,}142.02$$

b) We engage in a reverse cash and carry strategy. In particular, we do the following:

Description	Today	In 9 months
Long forward, resulting from customer purchase	0	$S_T - F_{0,T}$
Sell short the index	$+S_0$	$-S_T$
Lend $+S_0$	$-S_0$	$S_0 \times e^{rT}$
TOTAL	0	$S_0 \times e^{rT} - F_{0,T}$

Specifically, with the numbers of the exercise, we have:

Description	Today	In 9 months
Long forward, resulting from customer purchase	0	$S_T - \$1,142.02$
Sell short the index	$1,100	$-S_T$
Lend $ 1,100	−$1,100	$\$1,100 \times e^{0.05 \times 0.75}$ $= \$1,142.02$
TOTAL	0	0

Therefore, the market maker is perfectly hedged. She does not have any risk in the future, because she has successfully created a synthetic short position in the forward contract.

c) Now, we will engage in cash and carry arbitrage:

Description	Today	In 9 months
Short forward, resulting from customer purchase	0	$F_{0,T} - S_T$
Buy the index	$-S_0$	S_T
Borrow $+S_0$	$+S_0$	$-S_0 \times e^{rT}$
TOTAL	0	$F_{0,T} - S_0 \times e^{rT}$

Specifically, with the numbers of the exercise, we have:

Description	Today	In 9 months
Short forward, resulting from customer purchase	0	$\$1,142.02 - S_T$
Buy the index	−$1,100	S_T
Borrow $1,100	$1,100	$-\$1,100 \times e^{0.05 \times 0.75}$ $= -\$1,142.02$
TOTAL	0	0

Again, the market maker is perfectly hedged. He does not have any index price risk in the future, because he has successfully created a synthetic long position in the forward contract that perfectly offsets his obligation from the sold forward contract.

Question 5.6.

a) We plug the continuously compounded interest rate, the dividend yield and the time to expiration in years into the valuation formula and notice that the time to expiration is 9 months, or 0.75 years. We have:

$$F_{0,T} = S_0 \times e^{(r-\delta)\times T} = \$1,100 \times e^{(0.05-0.015)\times 0.75} = \$1,100 \times 1.0266 = \$1,129.26$$

b) We engage in a reverse cash and carry strategy. In particular, we do the following:

Description	Today	In 9 months
Long forward, resulting from customer purchase	0	$S_T - F_{0,T}$
Sell short tailed position of the index	$+S_0 e^{-\delta T}$	$-S_T$
Lend $S_0 e^{-\delta T}$	$-S_0 e^{-\delta T}$	$S_0 \times e^{(r-\delta)T}$
TOTAL	0	$S_0 \times e^{(r-\delta)T} - F_{0,T}$

Specifically, we have:

Description	Today	In 9 months
Long forward, resulting from customer purchase	0	$S_T - \$1,129.26$
Sell short tailed position of the index	$\$1,100 \times .9888 = 1087.69$	$-S_T$
Lend \$1,087.69	$-\$1,087.69$	$\$1,087.69 \times e^{0.05\times 0.75} = \$1,129.26$
TOTAL	0	0

Therefore, the market maker is perfectly hedged. He does not have any risk in the future, because he has successfully created a synthetic short position in the forward contract.

c)

Description	Today	In 9 months
Short forward, resulting from customer purchase	0	$F_{0,T} - S_T$
Buy tailed position in index	$-S_0 e^{-\delta T}$	S_T
Borrow $S_0 e^{-\delta T}$	$S_0 e^{-\delta T}$	$-S_0 \times e^{(r-\delta)T}$
TOTAL	0	$F_{0,T} - S_0 \times e^{(r-\delta)T}$

Specifically, we have:

Description	Today	In 9 months
Short forward, resulting from customer purchase	0	$\$1{,}129.26 - S_T$
Buy tailed position in index	$-\$1{,}100 \times .9888$ $= -\$1{,}087.69$	S_T
Borrow \$ 1,087.69	\$1,087.69	$-\$1{,}087.69 \times e^{0.05\times 0.75}$ $= -\$1{,}129.26$
TOTAL	0	0

Again, the market maker is perfectly hedged. He does not have any index price risk in the future, because he has successfully created a synthetic long position in the forward contract that perfectly offsets his obligation from the sold forward contract.

Question 5.7.

We need to find the fair value of the forward price first. We plug the continuously compounded interest rate and the time to expiration in years into the valuation formula and notice that the time to expiration is 6 months, or 0.5 years. We have:

$$F_{0,T} = S_0 \times e^{(r)\times T} = \$1{,}100 \times e^{(0.05)\times 0.5} = \$1{,}100 \times 1.02532 = \$1{,}127.85$$

a) If we observe a forward price of 1135, we know that the forward is too expensive, relative to the fair value we determined. Therefore, we will sell the forward at 1135, and create a synthetic forward for 1,127.85, make a sure profit of \$7.15. As we sell the real forward, we engage in cash and carry arbitrage:

Description	Today	In 9 months
Short forward	0	$\$1,135.00 - S_T$
Buy position in index	$-\$1{,}100$	S_T
Borrow \$1,100	$-\$1{,}100$	$\$1{,}127.85$
TOTAL	0	\$7.15

This position requires no initial investment, has no index price risk, and has a strictly positive payoff. We have exploited the mispricing with a pure arbitrage strategy.

b) If we observe a forward price of 1,115, we know that the forward is too cheap, relative to the fair value we have determined. Therefore, we will buy the forward at 1,115, and create a synthetic short forward for 1,127.85, make a sure profit of \$12.85. As we buy the real forward, we engage in a reverse cash and carry arbitrage:

Description	Today	In 9 months
Long forward	0	$S_T - \$1{,}115.00$
Short position in index	$\$1{,}100$	$-S_T$
Lend \$1,100	$-\$1{,}100$	$\$1{,}127.85$
TOTAL	0	\$12.85

This position requires no initial investment, has no index price risk, and has a strictly positive payoff. We have exploited the mispricing with a pure arbitrage strategy.

Question 5.8.

First, we need to find the fair value of the forward price. We plug the continuously compounded interest rate, the dividend yield and the time to expiration in years into the valuation formula and notice that the time to expiration is 6 months, or 0.5 years. We have:

$$F_{0,T} = S_0 \times e^{(r-\delta)\times T} = \$1{,}100 \times e^{(0.05-0.02)\times 0.5} = \$1{,}100 \times 1.01511 = \$1{,}116.62$$

a) If we observe a forward price of 1,120, we know that the forward is too expensive, relative to the fair value we have determined. Therefore, we will sell the forward at 1,120, and create a synthetic forward for 1,116.82, making a sure profit of \$3.38. As we sell the real forward, we engage in cash and carry arbitrage:

Description	Today	In 9 months
Short forward	0	$\$1{,}120.00 - S_T$
Buy tailed position in index	$-\$1{,}100 \times .99$ $= -\$1{,}089.055$	S_T
Borrow \$1,089.055	\$1,089.055	$-\$1{,}116.62$
TOTAL	0	\$3.38

This position requires no initial investment, has no index price risk, and has a strictly positive payoff. We have exploited the mispricing with a pure arbitrage strategy.

b) If we observe a forward price of 1,110, we know that the forward is too cheap, relative to the fair value we have determined. Therefore, we will buy the forward at 1,110, and create a synthetic short forward for 1116.62, thus making a sure profit of \$6.62. As we buy the real forward, we engage in a reverse cash and carry arbitrage:

Description	Today	In 9 months
Long forward	0	$S_T - \$1{,}110.00$
Sell short tailed position in index	$\$1{,}100 \times .99$ $= \$1{,}089.055$	$-S_T$
Lend \$1,089.055	$-\$1{,}089.055$	\$1,116.62
TOTAL	0	\$6.62

This position requires no initial investment, has no index price risk, and has a strictly positive payoff. We have exploited the mispricing with a pure arbitrage strategy.

Question 5.9.

a) A money manager could take a large amount of money in 1982, travel back to 1981, invest it at 12.5%, and instantaneously travel forward to 1982 to reap the benefits, i.e. the accured interest. Our argument of time value of money breaks down.

b) If many money managers undertook this strategy, competitive market forces would drive the interest rates down.

c) Unfortunately, these arguments mean that costless and riskless time travel will not be invented.

Question 5.10.

a) We plug the continuously compounded interest rate, the forward price, the initial index level and the time to expiration in years into the valuation formula and solve for the dividend yield:

$$
\begin{aligned}
& F_{0,T} = S_0 \times e^{(r-\delta)\times T} \\
\Leftrightarrow \quad & \frac{F_{0,T}}{S_0} = e^{(r-\delta)\times T} \\
\Leftrightarrow \quad & \ln\left(\frac{F_{0,T}}{S_0}\right) = (r-\delta) \times T \\
\Leftrightarrow \quad & \delta = r - \frac{1}{T}\ln\left(\frac{F_{0,T}}{S_0}\right) \\
\Rightarrow \quad & \delta = 0.05 - \frac{1}{0.75}\ln\left(\frac{1129.257}{1100}\right) = 0.05 - 0.035 = 0.015
\end{aligned}
$$

Remark: Note that this result is consistent with exercise 5.6., in which we had the same forward prices, time to expiration etc.

b) With a dividend yield of only 0.005, the fair forward price would be:

$$F_{0,T} = S_0 \times e^{(r-\delta)\times T} = 1{,}100 \times e^{(0.05-0.005)\times 0.75} = 1{,}100 \times 1.0343 = 1{,}137.759$$

Therefore, if we think the dividend yield is 0.005, we consider the observed forward price of 1,129.257 to be too cheap. We will therefore buy the forward and create a synthetic short forward, capturing a certain amount of \$8.502. We engage in a reverse cash and carry arbitrage:

Description	Today	In 9 months
Long forward	0	$S_T - \$1{,}129.257$
Sell short tailed position in index	$\$1{,}100 \times .99626$ $= \$1{,}095.88$	$-S_T$
Lend \$1,095.88	$-\$1{,}095.88$	\$1,137.759
TOTAL	0	\$8.502

c) With a dividend yield of 0.03, the fair forward price would be:

$$F_{0,T} = S_0 \times e^{(r-\delta)\times T} = 1{,}100 \times e^{(0.05-0.03)\times 0.75} = 1{,}100 \times 1.01511 = 1{,}116.62$$

Therefore, if we think the dividend yield is 0.03, we consider the observed forward price of 1,129.257 to be too expensive. We will therefore sell the forward and create a synthetic long forward, capturing a certain amount of \$12.637. We engage in a cash and carry arbitrage:

Description	Today	In 9 months
Short forward	0	$\$1{,}129.257 - S_T$
Buy tailed position in index	$-\$1{,}100 \times .97775$ $= -\$1{,}075.526$	S_T
Borrow \$1,075.526	\$1,075.526	\$1,116.62
TOTAL	0	\$12.637

Question 5.11.

a) The notional value of 4 contracts is $4 \times \$250 \times 1200 = \$1{,}200{,}000$, because each index point is worth \$250, and we buy four contracts.

b) The margin protects the counterparty against default. In our case, it is 10% of the notional value of our position, which means that we have to deposit an initial margin of:

$$\$1{,}200{,}000 \times 0.10 = \$120{,}000$$

Question 5.12.

a) The notional value of 10 contracts is $10 \times \$250 \times 950 = \$2{,}375{,}000$, because each index point is worth \$250, we buy 10 contracts and the S&P 500 index level is 950.

With an initial margin of 10% of the notional value, this results in an initial dollar margin of $\$2{,}375{,}000 \times 0.10 = \$237{,}500$.

b) We first obtain an approximation. Because we have a 10% initial margin, a 2% decline in the futures price will result in a 20% decline in margin. As we will receive a margin call after a 20% decline in the initial margin, the smallest futures price that avoids the maintenance margin call is $950 \times .98 = 931$. However, this calculation ignores the interest that we are able to earn in our margin account.

Let us now calculate the details. We have the right to earn interest on our initial margin position. As the continuously compounded interest rate is currently 6%, after one week, our initial margin has grown to:

$$\$237{,}500e^{0.06\times\frac{1}{52}} = \$237{,}774.20$$

We will get a margin call if the initial margin falls by 20%. We calculate 80% of the initial margin as:

$$\$237{,}500 \times 0.8 = \$190{,}000$$

10 long S&P 500 futures contracts obligate us to pay \$2,500 times the forward price at expiration of the futures contract.

Therefore, we have to solve the following equation:

$$\begin{aligned} & \$237{,}774.20 + (F_{1W} - 950) \times \$2{,}500 && \geq \$190{,}000 \\ \Leftrightarrow\ & \$47774.20 && \geq -(F_{1W} - 950) \times \$2{,}500 \\ \Leftrightarrow\ & 19.10968 - 950 && \geq -F_{1W} \\ \Leftrightarrow\ & F_{1W} && \geq 930.89 \end{aligned}$$

Therefore, the greatest S&P 500 index futures price at which we will receive a margin call is 930.88.

Question 5.13.

a)

Description	Today	At expiration of the contract
Long forward	0	$S_T - F_{0,T} = S_T - S_0e^{rT}$
Lend S_0	$-S_0$	S_0e^{rT}
Total	$-S_0$	S_T

In the first row, we made use of the forward price equation if the stock does not pay dividends. We see that the total aggregate position is equivalent to the payoff of one stock.

b) In the case of discrete dividends, we have:

Description	Today	At expiration of the contract
Long forward	0	$S_T - F_{0,T} = S_T - S_0 e^{rT} + \sum_{i=1}^{n} e^{r(T-t_i)} D_{t_i}$
Lend $S_0 - \sum_{i=1}^{n} e^{-rt_i} D_{t_i}$	$-S_0 + \sum_{i=1}^{n} e^{-rt_i} D_{t_i}$	$+S_0 e^{rT} - \sum_{i=1}^{n} e^{r(T-t_i)} D_{t_i}$
Total	$-S_0 + \sum_{i=1}^{n} e^{-rt_i} D_{t_i}$	S_T

c) In the case of a continuous dividend, we have to tail the position initially. We therefore create a synthetic share at the time of expiration T of the forward contract.

Description	Today	At expiration of the contract
Long forward	0	$S_T - F_{0,T} = S_T - S_0 e^{(r-\delta)T}$
Lend $S_0 e^{-\delta T}$	$-S_0 e^{-\delta T}$	$S_0 e^{(r-\delta)T}$
Total	$-S_0 e^{-\delta T}$	S_T

In the first row, we made use of the forward price equation if the stock pays a continuous dividend. We see that the total aggregate position is equivalent to the payoff of one stock at time T.

Question 5.14.

An arbitrageur believing that the observed forward price, F(0,T), is too low will undertake a reverse cash and carry arbitrage: Buy the forward, short sell the stock and lend out the proceeds from the short sale. The relevant prices are therefore the bid price of the stock and the lending interest rate. Also, she will incur the transaction costs twice. We have:

Description	Today	In 9 months
Long forward	0	$S_T - F_{0,T}$
Sell short tailed position of the index	$+S_0^b e^{-\delta T}$	$-S_T$
Pay twice transaction cost	$-2 \times k$	
Lend $S_0^b e^{-\delta T} - 2 \times k$	$-S_0^b e^{-\delta T} + 2 \times k$	$\left(+S_0^b e^{-\delta T} - 2 \times k\right) \times e^{r^l T}$
TOTAL	0	$\left(+S_0^b e^{-\delta T} - 2 \times k\right) \times e^{r^l T} - F_{0,T}$

To avoid arbitrage, we must have $\left(S_0^b - 2 \times k\right) \times e^{r^l T} - F_{0,T} \leq 0$. This is equivalent to $F_{0,T} \geq \left(S_0^b - 2 \times k\right) \times e^{r^l T}$. Therefore, for any $F_{0,T}$ smaller than this bound, there exist arbitrage opportunities.

Question 5.15.

a) We use the transaction cost boundary formulas that were developed in the text and in exercise 5.14. In this part, we set k equal to zero. There is no bid-ask spread. Therefore, we have

$$F^+ = 800e^{0.055} = 800 \times 1.05654 = 845.23$$
$$F^- = 800e^{0.05} = 800 \times 1.051271 = 841.02$$

b) Now, we will incur an additional transaction fee of \$1 for going either long or short the forward contract. Stock sales or purchases are unaffected. We calculate:

$$F^+ = (800 + 1)\, e^{0.055} = 801 \times 1.05654 = 846.29$$
$$F^- = (800 - 1)\, e^{0.05} = 799 \times 1.051271 = 839.97$$

c) Now, we will incur an additional transaction fee of \$2.40 for the purchase or sale of the index, making our total initial transaction cost \$3.40. We calculate:

$$F^+ = (800 + 3.40)\, e^{0.055} = 803.40 \times 1.05654 = 848.82$$
$$F^- = (800 - 3.40)\, e^{0.05} = 796.60 \times 1.051271 = 837.44$$

d) We also have to take into account as well the additional cost that we incur at the time of expiration. We can calculate:

$$F^+ = (800 + 3.40)\, e^{0.055} + 2.40 = 803.40 \times 1.05654 = 851.22$$
$$F^- = (800 - 3.40)\, e^{0.05} - 2.40 = 796.60 \times 1.051271 = 835.04$$

e) Let us make use of the hint. In the cash and carry arbitrage, we will buy the index and have thus at expiration time S_T. However, we have to pay a proportional transaction cost of 0.3% on it, so that the position is only worth $0.997 \times S_T$. However, we need S_T to set off the index price risk introduced by the short forward. Therefore, we will initially buy 1.003 units of the index, which leaves us exactly S_T after transaction costs. Additionally, we incur a transaction cost of $0.003 \times S_0$ for buying the index today, and of \$1 for selling the forward contract.

$$F^+ = (800 \times 1.003 + 800 \times 0.003 + 1)\, e^{0.055} = 805.80 \times 1.05654 = 851.36$$

The boundary is slightly higher, because we must take into account the variable, proportional cash settlement cost we incur at expiration. The difference between part d) and part e) is the interest we have to pay on \$2.40, which is \$.14.

In the reverse cash and carry arbitrage, we will sell the index and have to pay back at expiration $-S_T$. However, we have to pay a proportional transaction cost of 0.3% on it, so that we have exposure of $-1.003 \times S_T$. However, we only need an exposure of $-S_T$ to set off the index price risk introduced by the long forward. Therefore, we will initially only sell 0.997 units of the index, which leaves us exactly with $-S_T$ after transaction costs at expiration. Additionally, we incur a transaction cost of $0.003 \times S_0$ for buying the index today, and \$1 for selling the forward contract. We have as a new lower bound:

$$F^{-} = (800 \times 0.997 - 800 \times 0.003 - 1)\, e^{0.05} = 794.20 \times 1.051271 = 834.92$$

The boundary is slightly lower, because we forego some interest we could earn on the short sale, because we have to take into account the proportional cash settlement cost we incur at expiration. The difference between part d) and part e) is the interest we are foregoing on \$2.40, which is \$.12 (at the lending rate of 5%).

Question 5.16.

a) The one-year futures price is determined as:

$$F_{0,1} = 875e^{0.0475} = 875 \times 1.048646 = 917.57$$

b) One futures contract has the value of $\$250 \times 875 = \$218{,}750$. Therefore, the number of contracts needed to cover the exposure of \$800,000 is: $\$800{,}000 \div \$218{,}750 = 3.65714$. Furthermore, we need to adjust for the difference in beta. Since the beta of our portfolio exceeds 1, it moves more than the index in either direction. Therefore, we must increase the number of contracts. The final hedge quantity is: $3.65714 \times 1.1 = 4.02286$. Therefore, we should short-sell 4.02286 S&P 500 index future contracts.

As the correlation between the index and our portfolio is assumed to be one, we have no basis risk and have perfectly hedged our position and transformed it into a riskless investment. Therefore, we expect to earn the risk-free interest rate as a return over one year.

Question 5.17.

It is important to realize that, because we can go long or short a future, the sign of the correlation does not matter in our ranking. Suppose the correlation of our portfolio in question 5.16. with the S&P 500 is minus 1. Then we can do exactly the same calculation, but would in the end go long the futures contract.

It is thus the absolute correlation coefficient that should be as close to one as possible. Therefore, the ranking is 0, 0.25, −0.5, −0.75, 0.85, −0.95, with 0 having the highest basis risk.

Question 5.18.

The current exchange rate is 0.02E/Y, which implies 50Y/E. The euro continuously compounded interest rate is 0.04, the yen continuously compounded interest rate 0.01. Time to expiration is 0.5 years. Plug the input variables into the formula to see that:

$$\text{Euro/Yen forward} = 0.02e^{(0.04-0.01)\times 0.5} = 0.02 \times 1.015113 = 0.020302$$
$$\text{Yen/Euro forward} = 50e^{(0.01-0.04)\times 0.5} = 50 \times 0.98511 = 49.2556$$

Question 5.19.

The current spot exchange rate is 0.008$/Y, the one-year continuously compounded dollar interest rate is 5%, and the one-year continuously compounded yen interest rate is 1%. This means that we can calculate the fair price of a one-year $/Yen forward to be:

$$\text{Dollar/Yen forward } = 0.008e^{(0.05-0.01)} = 0.008 \times 1.0408108 = 0.0083265$$

We can see that the observed forward exchange rate of 0.0084 $/Y is too expensive, relative to the fair forward price. We therefore sell the forward and synthetically create a forward position:

Description	Today		At expiration	of the contract
	in $	in Yen	in $	in Yen
Sell $/Y forward	0	—	0.0084$	-1
Buy Yen for 0.0079204 dollar	-0.0079204	$+0.99005$	—	—
Lend 0.99005 Yen	—	-0.99005	—	1
Borrow 0.0079204 dollar	$+0.0079204$	—	-0.0083265	—
Total	0	0	0.0000735	0

Therefore, this transaction earned us 0.0000735 dollars, without any exchange risk or initial investment involved. We have exploited an inherent arbitrage opportunity.

With a forward exchange rate of 0.0083, the observed price is too cheap. We will buy the forward and synthetically create a short forward position.

Description	Today		At expiration	of the contract
	in $	in Yen	in $	in Yen
Buy $/Y forward	0	—	$-0.0083$$	$+1$
Sell Yen for 0.0079204 dollar	$+0.0079204$	-0.99005	—	—
Borrow 0.99005 Yen	—	$+0.99005$	—	-1
Lend 0.0079204 dollar	-0.0079204	—	$+0.0083265$	—
Total	0	0	0.0000265	0

Therefore, we again made an arbitrage profit of 0.0000265 dollars.

Question 5.20.

a) The Eurodollar futures price is 93.23. Therefore, we can use equation (5.20) of the main text to back out the three-month LIBOR rate:

$$r_{91} = (100 - 93.23) \times \frac{1}{100} \times \frac{1}{4} \times \frac{91}{90} = 0.017113.$$

b) We will have to repay principal plus interest on the loan that we are taking from the following June to September. Because we shorted a Eurodollar futures, we are guaranteed the interest rate we calculated in part a). Therefore, we have a repayment of:

$$\$10{,}000{,}000 \times (1 + r_{91}) = \$10{,}000{,}000 \times 1.017113 = \$10{,}171{,}130$$