American Options
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• The value of the option if it is left “alive” (i.e., unexercised) is given by the value of holding it for another period, equation (10.3)

• The value of the option if it is exercised is given by $\max(0, S - K)$ if it is a call and $\max(0, K - S)$ if it is a put

• For an American call, the value of the option at a node is given by

$$C(S, K, t) = \max(S = K, e^{-rh}[C(uS, K, t + h)p^* + C(dS, K, t + h)(1 - p^*)])$$
American Options (cont’d)

• The valuation of American options proceeds as follows
  – At each node, we check for early exercise
  – If the value of the option is greater when exercised, we assign that value to the node. Otherwise, we assign the value of the option unexercised
  – We work backward through the three as usual
American Options (cont’d)

- American put option

\[
\begin{align*}
\text{Value at } S = 30.585: & \quad \Delta = -1.000 \quad B = $38.947 \\
\text{Value at } S = 35.411: & \quad \Delta = -0.633 \quad B = $28.018 \\
\text{Value at } S = 41.000: & \quad \Delta = -0.332 \quad B = $16.891 \\
\text{Value at } S = 50.071: & \quad \Delta = -0.078 \quad B = $0.000 \\
\text{Value at } S = 61.149: & \quad \Delta = 0.000 \quad B = $0.000 \\
\end{align*}
\]
American Options (cont’d)

• The only difference in the binomial tree occurs at the $S_{dd}$ node, where the stock price is $30.585. The American option at that point is worth $40 – $30.585 = $9.415, its early-exercise value (as opposed to $8.363 if unexercised). The greater value of the option at that node ripples back through the tree.

• Thus, an American option is more valuable than the otherwise equivalent European option.
Understanding Early Exercise
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- Options may be rationally exercised prior to expiration

- By exercising, the option holder
  - receives the stock and thus receives dividends
  - pays the strike price prior to expiration (this has an interest cost)
  - loses the insurance implicit in the call against the possibility that the stock price will be less than the strike price at expiration
Understanding Early Exercise

• If volatility is zero, the value of insurance is zero. Then, it is optimal to defer exercise as long as interest savings on the strike exceed dividends lost

\[ rK > \delta S \]

• Therefore, it is optimal to exercise when

\[ S > \frac{rK}{\delta} \]

In the special case when \( r = \delta \) and \( \sigma = 0 \), any in-the-money option should be exercised immediately

• When volatility is positive, the implicit insurance has value, and the value varies with time to expiration
Understanding Early Exercise (Cont’d)

- The following graph displays the price, above which early exercise is optimal for a 5-year call option with $K = $100, $r = 5\%$, and $\delta = 5\%$
Understanding Early Exercise (Cont’d)

- The following graph displays the price, above which early exercise is optimal for a 5-year put option with $K = $100, $r = 5\%$, and $\delta = 5\%$. 

![Exercise Boundary Graph](image)