# American Options

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- The value of the option if it is left “alive” (i.e., unexercised) is given by the value of holding it for another period, equation (10.3)
- The value of the option if it is exercised is given by *max* (0, *S* – *K*) if it is a call and *max* (0, *K* – *S*) if it is a put
- For an American call, the value of the option at a node is given by

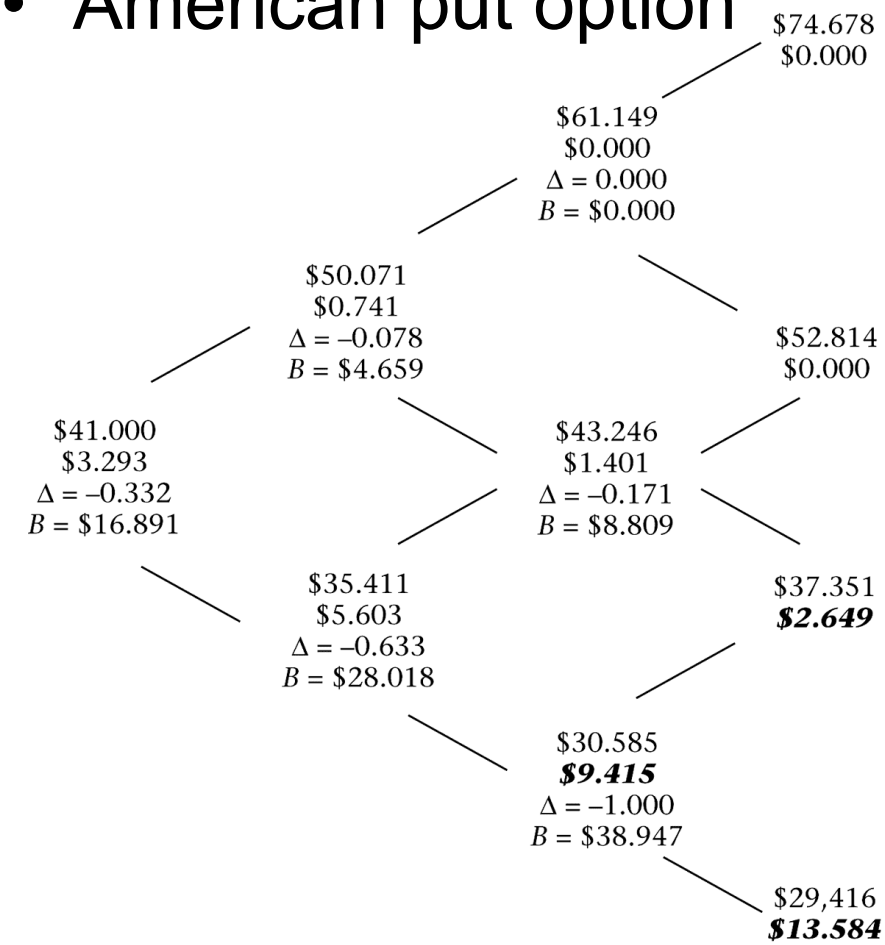
$$C(S,K,t) = \max(S = K, e^{-rh}[C(uS,K,t+h)p^* + C(dS,K,t+h)(1-p^*)])$$

# American Options (cont'd)

- The valuation of American options proceeds as follows
  - At each node, we check for early exercise
  - If the value of the option is greater when exercised, we assign that value to the node. Otherwise, we assign the value of the option unexercised
  - We work backward through the three as usual

# American Options (cont'd)

- American put option



# American Options (cont'd)

- The only difference in the binomial tree occurs at the $S_{dd}$ node, where the stock price is \$30.585. The American option at that point is worth \$40 – \$30.585 = \$9.415, its early-exercise value (as opposed to \$8.363 if unexercised). The greater value of the option at that node ripples back through the tree
- Thus, an American option is more valuable than the otherwise equivalent European option

# Understanding Early Exercise

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- Options may be rationally exercised prior to expiration
- By exercising, the option holder
  - receives the stock and thus receives dividends
  - pays the strike price prior to expiration (this has an interest cost)
  - loses the insurance implicit in the call against the possibility that the stock price will be less than the strike price at expiration

# Understanding Early Exercise

- If volatility is zero, the value of insurance is zero. Then, it is optimal to defer exercise as long as interest savings on the strike exceed dividends lost

$$rK > \delta S$$

- Therefore, it is optimal to exercise when

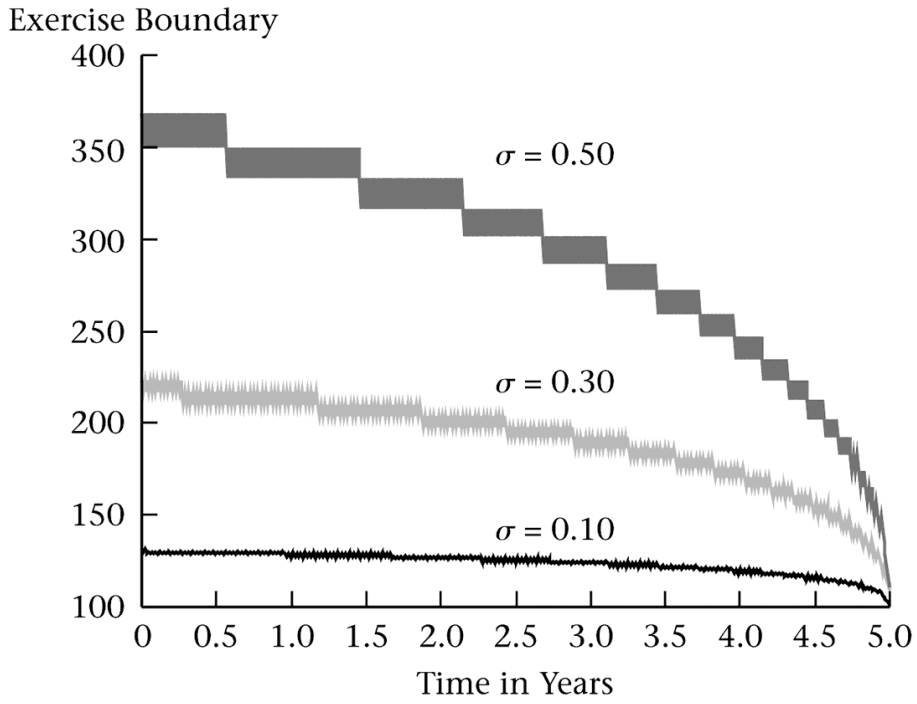
$$S > \frac{rK}{\delta}$$

In the special case when r = $\delta$ and $\sigma$ = 0, any in-the-money option should be exercised immediately

- When volatility is positive, the implicit insurance has value, and the value varies with time to expiration

# Understanding Early Exercise (Cont'd)

- The following graph displays the price, above which early exercise is optimal for a 5-year call option with $K$ = \$100, $r$ = 5%, and $\delta$ = 5%



# Understanding Early Exercise (Cont'd)

- The following graph displays the price, above which early exercise is optimal for a 5-year put option with $K$ = \$100, $r$ = 5%, and $\delta$ = 5%