Caplet

\[ R_{k-1}(k-1, k) \]

\[ k-1 \quad k \]

OLB = L

\[ L \times R_{k-1}(k-1, k) \]

the original amt

You wish to pay at most \[ L \times K_e \]

"strike" interest rate aka

the CAP RATE

\[ (L \times R_{k-1}(k-1, k) - L \times K_e)_+ = \]

\[ = L \times (R_{k-1}(k-1, k) - K_e)_+ \quad @ \text{time } k. \]

*in arrears*

If the payment is made @ time \((k-1)\) instead:

\[ \frac{L \times (R_{k-1}(k-1, k) - K_e)_+}{1 + R_{k-1}(k-1, k)} \]

*in advance*

Cap ... a bundle of caplets (one for every loan repayment installment).
Q: What is the net-effect for the borrower @ time-\(k\)?

\[
-L \times R_{k-1}(k-1,k) + L \times (R_{k-1}(k-1,k) - K_F)_+ = -L \times \min(R_{k-1}(k-1,k), K_F)
\]

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**Binomial interest-rate trees**

![](image)

**Example.** Price of a zero-coupon bond redeemable @ time-2 for $1.

![Price diagram](image)

Interest rates effective p... risk-neutral probability needs to be exogenously prescribed
\[ P(0,2) = \frac{1}{1+r_0} \times \frac{1}{1+r_{ru}} \times P + \frac{1}{1+r_0} \times \frac{1}{1+r_{rd}} \times (1-P) \]

\[ P(0,2) = \frac{1}{1+r_0} \left[ P \times \frac{1}{1+r_{ru}} + (1-P) \times \frac{1}{1+r_{rd}} \right] \]

By def'n

Most commonly, \( p = \frac{1}{2} \) (the tree is symmetric).

\[ P(0,2) = P(0,1) \times \frac{1}{2} \times \left[ \frac{1}{1+r_{ru}} + \frac{1}{1+r_{rd}} \right] \]

\[ P(0,2) \times P_u = \frac{1}{1+r_{ru}} \]

\[ P(0,2) \times P_d = \frac{1}{1+r_{rd}} \]

Lecture note: Example 1.2.