M329F: February 19th, 2020.

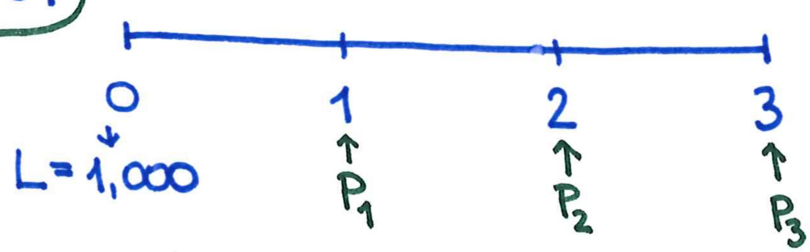
PROBLEM. [EARLY REPAYMENT]

You borrow \$1,000 @ an annual effective interest rate of 4% and agree to repay the loan in 3 annual installments @ the end of every year.
The amount of each payment in the last two years is set @ twice that in the first year.

At the end of one year, you have the option to repay the entire loan w/ a pmt of X in addition to the regular pmt.
This must yield the lender an annual effective interest rate of 4.5%. Find X.

→: In the original scheme:

$i=0.04$



1.

w/ $\quad 2P_1 = P_2 = P_3$

NPV(The three pmts @ rate $i=0.04$) = 1,000,

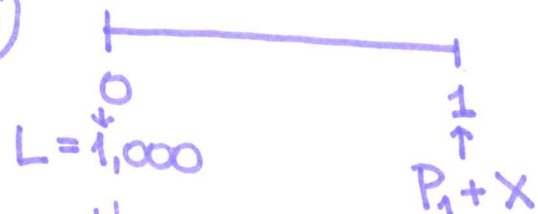
i.e., $\quad P_1 \cdot (1+i)^{-1} + P_2(1+i)^{-2} + P_3(1+i)^{-3} = 1{,}000$

$$P_1(1.04)^{-1} + 2P_1(1.04)^{-2} + 2P_1(1.04)^{-3} = 1000$$

$$\Rightarrow \boxed{P_1 = \frac{1000}{(1.04)^{-1} + 2(1.04)^{-2} + 2(1.04)^{-3}} = 217.9293}$$

If there is early repayment:

$y = 0.045$

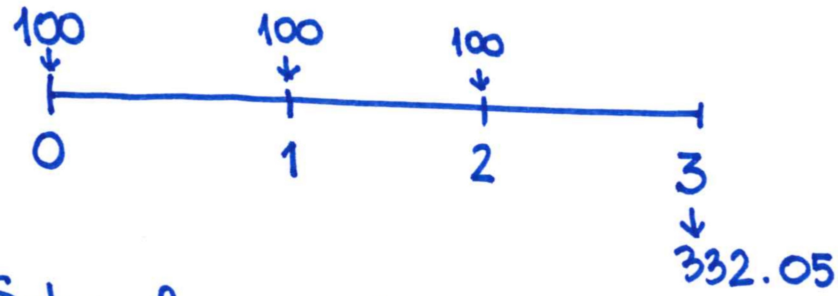


Under the requirement for this yield rate for the lender, we have:

$$L(1+y)^1 = P_1 + X$$

$$\Rightarrow X = 1{,}000(1.045) - 217.9293$$
$$= 1{,}045 - 217.9293 = 827.07$$

Roger's yield rate from reinvestment



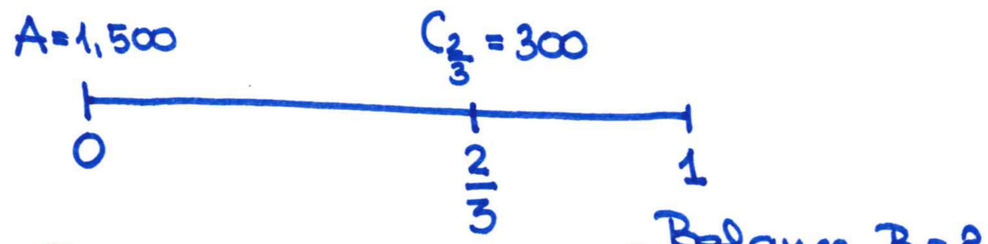
Solve for y in

$$100(1+y)^3 + 100(1+y)^2 + 100(1+y) = 332.05$$

Problem.

A fund began the year w/ \$1,500. A deposit of \$300 was made on September 1st of the same year.
Find the fund balance @ the end of that year if the dollar-weighted annual effective yied equals 12.5%.

→:



Given $j = 12.5\%$

We know: $B = A + C_{\frac{2}{3}} + I =$

$= 1500 + 300 + I$

$I = j \cdot \left(1500 \cdot 1 + 300\left(\frac{1}{3}\right)\right) =$

$I = 0.125(1600) = 200$

$B = 2000$ ∎