M329F: February 24th, 2020.

May 2001. Course 2.

31. You are given the following information about an investment account:

Date	Value Immediately Before Deposit	Deposit
January 1	10	
July 1	12	X
December 31	X	

Over the year, the time-weighted return is 0%, and the dollar-weighted return is ~~Y~~ y.

Calculate ~~Y~~ y.

(A) –25%

(B) –10%

(C) 0%

(D) 10%

(E) 25%

A... initial amt

C... contribution

B... the final balance

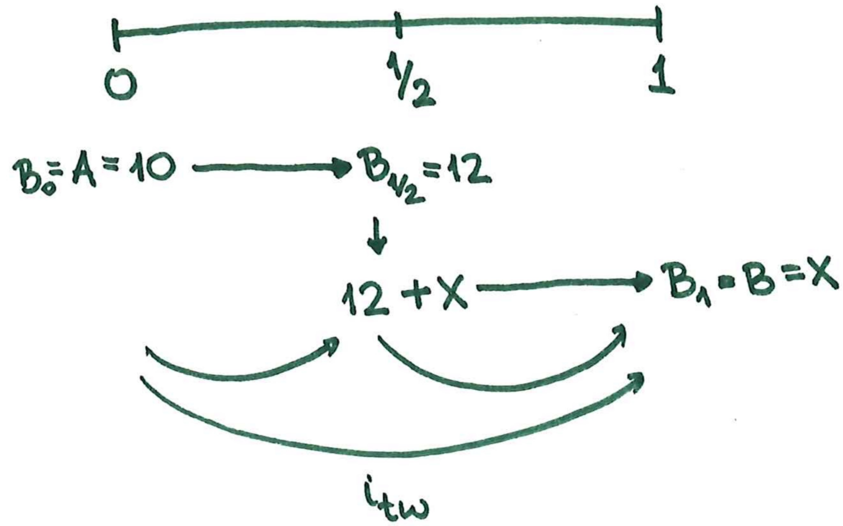
$$A + \underset{\substack{\uparrow \\ \text{interest}}}{I} + C = B$$

$$\left.\begin{aligned} A &= 10 \\ C &= X \\ B &= X \end{aligned}\right\} \quad I = B - A - C = X - 10 - X = -10$$

$$y = \frac{I}{A(1) + X(\frac{1}{2})} = \frac{-10}{10 + X(\frac{1}{2})}$$

We get the unknown X from the time-weighted return $i_{tw} = 0$.

1.



$$1 + i_{tw} = \frac{12}{10} \cdot \frac{X}{12 + X} = 1$$

$$12X = 10(12 + X)$$

$$12X = 120 + 10X$$

$$2X = 120$$

$$\boxed{X = 60}$$

$$y_{DW} = \frac{-10}{10 + 60\left(\frac{1}{2}\right)} = -\frac{10}{40} = -0.25$$

Problem. Tony just won the actuarial lottery. He is given the following three payout options:

(i) \$575,000 payable 6 months from today;

(ii) \$300,000 today plus an additional \$300,000 payable two years from today;

(iii) \$20,000 paid @ the beginning of each six month period forever w/ 1st payment today.

Assume $i^{(2)} = 0.08$

Find the option w/ the highest net present value.

→: j... the effective half·year interest rate

$\Rightarrow j = 0.04$

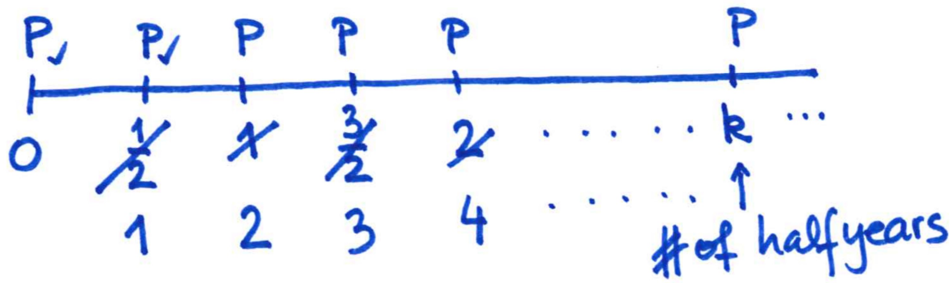
$\Rightarrow$ the discount factor per half·year period

$$v_j = \frac{1}{1+j} = \frac{1}{1.04}$$

(i) $575{,}000 \cdot v_j = 552{,}884.62$.

(ii) $300{,}000 + 300{,}000 \cdot v_j^4 = 556{,}441.26$

(iii) Set $P = 20{,}000$



$$NPV(iii) = P + P\cdot v_j^1 + P\cdot v_j^2 + \cdots + P\cdot v_j^k + \cdots$$

$$= P\left(1 + v_j + v_j^2 + \cdots + v_j^k + \cdots\right)$$

$$= P\cdot \frac{1}{1-v_j}$$

Sum of a geometric series

$|v_j| < 1$

$\Downarrow$

converges

$$\frac{1}{1-v_j} = \frac{1}{1-\frac{1}{1+j}} = \frac{1}{\frac{\cancel{1}+j-\cancel{1}}{1+j}} = \frac{1+j}{j}$$

$\Rightarrow$ In our problem:

$$NPV(iii) = 20{,}000 \cdot \frac{1.04}{0.04} = 520{,}000$$

We would choose (ii).