Consider an account whose balance grows so that THE RATE OF CHANGE is proportional to the current balance in the account.
Denote the constant of proportionality by δ.
Say :
- b_0 ... the initial balance in the account
- $b(t)$ ... the balance @ time t in my account

By my assumptions:

$$b'(t) = \frac{db(t)}{dt} = \delta\, b(t)$$

$$db(t) = \delta\, b(t)\, dt$$

$$d(\ln(b(t))) = \frac{db(t)}{b(t)} = \delta\, dt$$

$$\ln(b(t)) = \delta t + c$$

$$b(t) = e^c \cdot e^{\delta t}$$

Use $b(0) = b_0 = e^c$:

Finally : $b(t) = b_0 e^{\delta t}$

$\Rightarrow$ The accumulation f'tion : $a(t) = e^{\delta t}$

PROBLEM. [COMPARISON OF RATES]

Roger has \$1,000 to invest.
Which of the following will get him the greatest amt of interest after one year?

(a) simple annual interest rate of 9%
(b) annual effective interest rate of 8.25%
(c) nominal discount rate convertible quarterly of 8.75%
(d) nominal interest rate convertible semiannually of 8.5%
(e) force of interest of 7.75%

→ : Let's calculate the balances:

X (a) $1{,}000 \cdot (1+0.09\cdot(1)) = 1090$ X

X (b) $1{,}000 \cdot (1+0.0825)^1 = 1082.50$ X

(c) $$1{,}000\left(1-\frac{d^{(4)}}{4}\right)^{-4} = 1{,}000\left(1-\frac{0.0875}{4}\right)^{-4}$$
$$= 1092.50$$

X (d) $$1{,}000\left(1+\frac{i^{(2)}}{2}\right)^{2} = 1{,}000\left(1+\frac{0.085}{2}\right)^{2}$$
$$= 1{,}086.81$$ X

X (e) $1{,}000\, e^{0.0775} = 1{,}080.58$ X

76.

Consider two 30-year bonds with the same purchase price. Each has an annual coupon rate of 5% paid semiannually and a par value of 1000.

The first bond has an annual nominal yield rate of 5% compounded semiannually, and a redemption value of 1200.

The second bond has an annual nominal yield rate of j compounded semiannually, and a redemption value of 800.

Calculate j.

(A) 2.20%

(B) 2.34%

(C) 3.53%

(D) 4.40%

(E) 4.69%

* 77.

Lucas opens a bank account with 1000 and lets it accumulate at an annual nominal interest rate of 6% convertible semiannually. Danielle also opens a bank account with 1000 at the same time as Lucas, but it grows at an annual nominal interest rate of 3% convertible monthly.

* For each account, interest is credited only at the end of each interest conversion period.

* Calculate the number of months required for the amount in Lucas's account to be at least double the amount in Danielle's account.

(A) 276

(B) 282

(C) 285

(D) 286

(E) 288

3.

→ : • Focus on Lucas: his balance @ time t is

$$B_L(t) = 1000\left(1 + \frac{i^{(2)}}{2}\right)^{2t}$$

$$= 1000\left(1 + \frac{0.06}{2}\right)^{2t}$$

$$= 1000\,(1.03)^{2t}$$

• Focus on Danielle: her balance @ time t is

$$B_D(t) = 1000\left(1 + \frac{i^{(12)}}{12}\right)^{12t}$$

$$= 1000\left(1 + \frac{0.03}{12}\right)^{12t}$$

$$= 1000\,(1.0025)^{12t}$$

* ⇒ $$B_L(t) = 2B_D(t)$$

$$\cancel{1000}(1.03)^{2t} = 2\cdot\cancel{1000}(1.0025)^{12t} \qquad /\ln$$

$$2t\cdot\ln(1.03) = \ln(2) + 12t\ln(1.0025)$$

$$t\left(2\ln(1.03) - 12\ln(1.0025)\right) = \ln(2)$$

$$t = \frac{\ln(2)}{2\ln(1.03) - 12\ln(1.0025)} = 23.775$$

* ⇒ Our answer needs to be in months and a multiple of six.

$24 \times 12 = 288$ ■

⇒ (E)