

*Note:* You **must** show all your work. Numerical answers without a proper explanation or a clearly written down path to the solution will be assigned zero points.

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Please, provide your complete solution to the following problems:

**Problem 5.1.** (5 points) A perpetuity is payable continuously at the annual rate of  $1 + t^2$  at time  $t$ . Assume that the force of interest is constant at  $\delta = 0.05$ . Find the present value of the perpetuity.

**Problem 5.2.** (5 points) Solve problem **4.6.2.** from the textbook.

**Problem 5.3.** (5 points) Solve problem **4.Review.2.** from the textbook.

**Problem 5.4.** (10 points) Solve problem **5.2.2.** from the textbook.

**Problem 5.5.** (10 points) Assume that compound interest is used and denote the annual discount factor by  $v$ . Find the expression for the present value of a perpetuity which pays 1 at the end of the third year, 3 at the end of the sixth year, 5 at the end of the ninth year etc. The expression you get should be a function of the discount factor  $v$ .

**Problem 5.6.** (5 points) Roger buys a perpetuity-immediate with annual payments. The first payment is equal to 100, and subsequent payments increase by 10.

Harry has a level perpetuity-due with annual payments of 200.

Under an unknown interest rate  $i > 0$ , the present values of the above perpetuities are equal. Find  $i$ .

**Problem 5.7.** (10 pts) Roger's aunt Agatha intends to invest a level amount of money equal to \$1,000 into a fund at the end of every year for 20 years. This fund is projected to earn an annual effective interest rate of 10%. At the end every year, aunt Agatha will have to withdraw the interest accrued during that year and reinvest it at an annual effective rate of 6%.

Assume that aunt Agatha wants the annual yield rate of the above investment scheme to be 8% over the 20-year life of the investment.

How much should she charge Roger today for the right to the above investment in order to get the yield she wants?