

Note: You **must** show all your work. Numerical answers without a proper explanation or a clearly written down path to the solution will be assigned zero points.

Problem 6.1. (5 points) Roger's aunt Agatha decided to set up a monthly supplemental income for her favourite nephew (Roger, of course). Roger is supposed to get 1,000 at the end of each month for the first year. Then, for the second year his monthly allowances increase to 1,200. The same increase happens each year for as long as Roger receives this allowance. So, the amount he gets at the end of every month during year j is equal to

$$1,000 + 200(j - 1) = 800 + 200j.$$

Roger is supposed to be receiving the payments for 20 years.

To make this possible aunt Agatha opened an account with the annual effective interest rate equal to 0.05. She made one initial deposit A which was just sufficient to finance Roger's allowance. Find A .

Problem 6.2. (10 points) Solve problem **5.4.2.** from the textbook.

Problem 6.3. (10 points) Solve problem **5.Review.2** from the textbook.

Problem 6.4. (5 points) *Source: Sample FM Exam # 21.*

Consider a continuous annuity whose payments are made at a rate

$$f(t) = (8 + t)k, 0 \leq t \leq 10$$

where k is an unknown constant.

The annuity's payments are made to an account governed by the rate of interest $\delta_t = 1/(8+t)$.

There are no other deposits or withdrawals to this account. At time 10, the balance in the account equals 20,000. Find k .

Problem 6.5. (5 points) Find the present value of a continuous annuity with a term of $n = 10$ years if

- the time varying force of interest is given to be $\delta_t = (1 + t)^{-1}$, **and**
- the rate of payment at time $t \in [0, n]$ is $(1 + t)^2$ per annum.

Problem 6.6. (5 points) Solve problem **6.2.6** from the textbook.

Problem 6.7. (5 points) Solve problem **6.3.2** from the textbook.

Problem 6.8. (5 points) Solve problem **6.5.4** from the textbook.