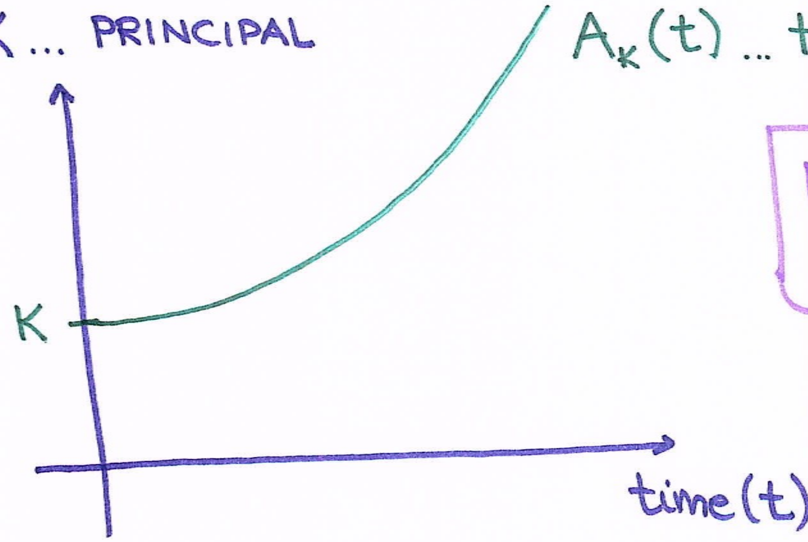
Review:

K ... PRINCIPAL

$A_K(t)$ ... the **amount** function

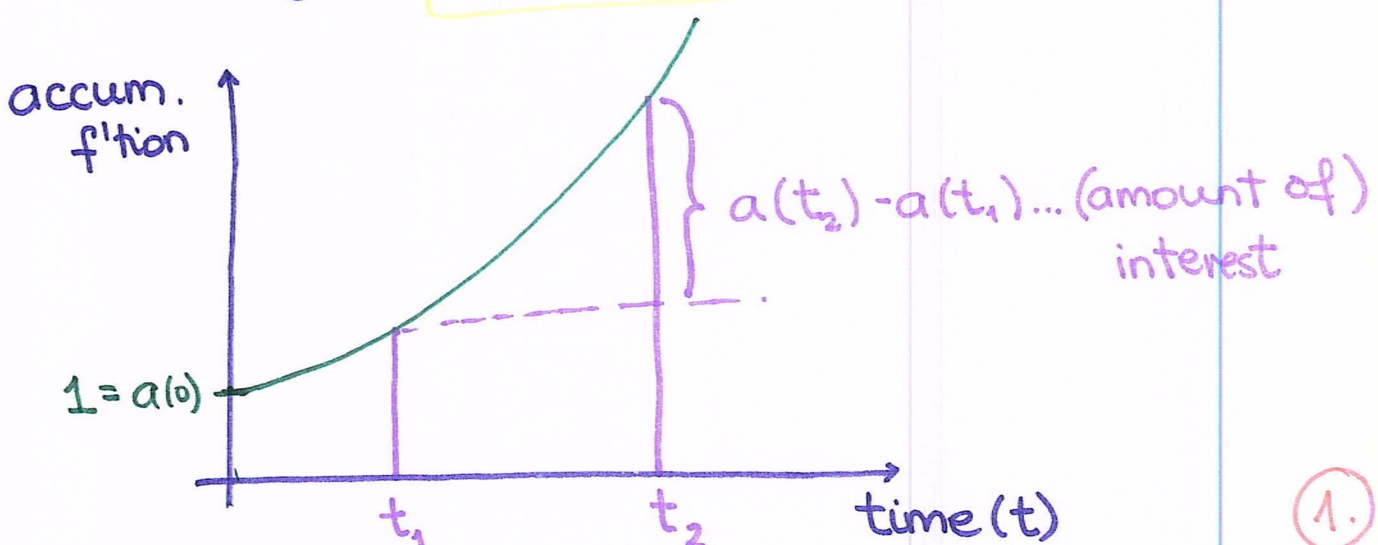


M329F: January 27th, 2020.

We introduce the **accumulation** f'tion $a(\cdot)$ which corresponds to \$1 principal

($\cdot$ ↑ time)

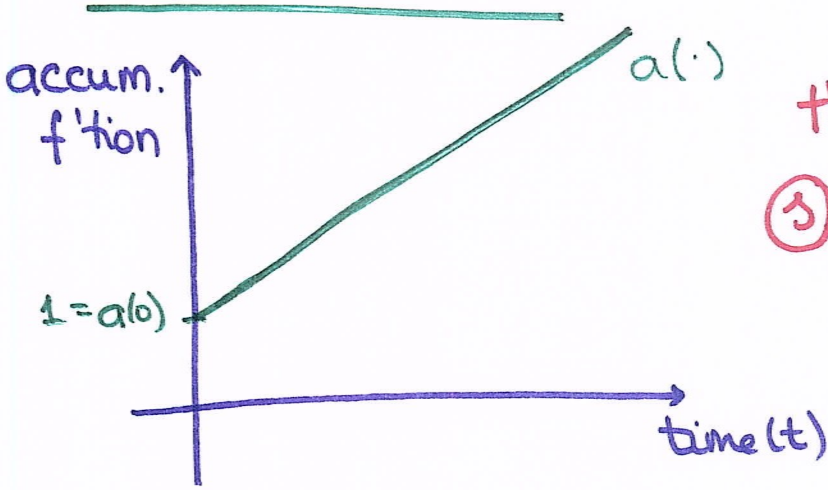
Usually: $\boxed{A_K(t) = K \cdot a(t)}$



1.

$$i_{[t_1,t_2]} := \frac{a(t_2) - a(t_1)}{a(t_1)}$$ ... effective interest rate during $[t_1, t_2]$

SIMPLE INTEREST



the slope of the line is

s .. simple interest rate

$\Downarrow$

$$a(t) = 1 + s\,t$$

M329F Theory of Interest
University of Texas at Austin
Problem set I
Instructor: Milica Čudina

1. (5 points) *Source: Problem 1.3.2 from the textbook.*
If you invest \$2,000 at time 0 and the accumulation function is given by

$$a(t) = 1 + 0.04t \qquad t \geq 0,$$

how much will you have at time 5?

→ :
$$a(5) = 1 + 0.04(5) = 1.2$$
$$2000(1.2) = 2400 \quad \blacksquare$$

2. (5 points) *Source: Problem 1.4.6(a) from the textbook.* $s = 0.05$
A loan is made at time 0 at simple interest at a simple interest rate of 0.05. For a certain $n \in \mathbb{N} = \{1, 2, 3, \dots\}$, the effective interest rate i_n turns out to be equal to 1/23. Find n.

$$i_n = i_{[n-1,n]} = \frac{a(n) - a(n-1)}{a(n-1)} =$$

$$= \frac{\cancel{1} + \cancel{s \cdot n} - (\cancel{1} + s(\cancel{n}-1))}{a(n-1)}$$

$$= \frac{s}{1 + s(n-1)}$$

In our problem:

$$\frac{1}{23} = \frac{0.05}{1 + 0.05(n-1)}$$

$$1 + 0.05(n-1) = 0.05 \cdot 23 \qquad / \cdot 20$$

$$20 + (n-1) = 23$$

$$\Rightarrow \boxed{n = 4} \quad \blacksquare$$

3.

Problem. [TWO SIMPLE INTEREST ACCOUNTS]

At a certain rate of simple interest, 1000 will accumulate to 1100 after a certain period of time.

Find the amount to which 500 accumulates in a different acct which is also governed by simple interest but @ a rate 3/4 as great as the one in the first acct and for the period twice as long.

→ : $\begin{cases} T \ldots \text{the length of time for } 1^{st} \text{ acct} \\ s \ldots \text{the simple interest rate for } 1^{st} \text{ acct} \end{cases}$

$$\Rightarrow \quad 1000(1+s\cdot T) = 1100 \quad \Rightarrow \boxed{s\cdot T = 0.1}$$

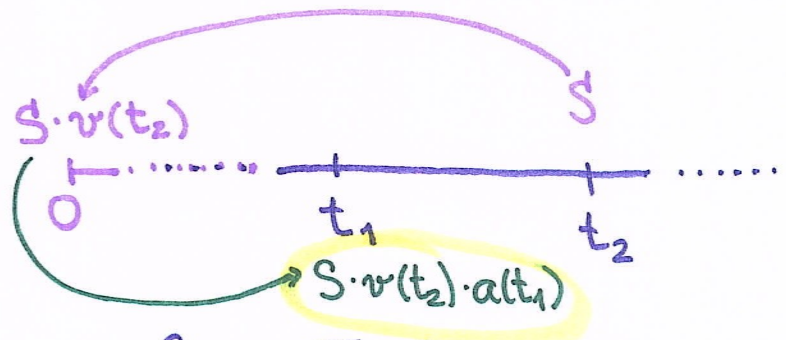
• for the second acc't:

$$500\left(1+\left(\frac{3}{\cancel{4}_2}\cdot s\right)(\cancel{2}T)\right) = \cancel{x}$$

$$= 500\left(1+\frac{3}{2}(s\cdot T)\right) = 500(1+0.15)$$

$$= 575$$

4.



<u>Goal</u>: To have S in the account @ time t_2.

Q: How much, in terms of S and $a(\cdot)$, do I need to deposit @ time t_1?

$$S v(t_2) \cdot a(t_1) = S \cdot \frac{a(t_1)}{a(t_2)} = S \cdot \frac{v(t_2)}{v(t_1)}$$

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Problem 2.1. (2 pts) Let the amount function A_K have the form

$$A_K(t) = \alpha(t+1)^3 + \beta(t+1).$$

Then, we know that $\alpha + \beta = K$. *True or false?*

Problem 2.2. (2 pts) If we wish to invest the amount X at a future time t_1 in order to have \$S at time $t_2 > t_1$, we should invest $X = S\,\frac{a(t_2)}{a(t_1)}$. *True or false?*

Problem 2.3. *Source: Problem 1.5.4 from the textbook.*
How much interest is earned in the fourth year by \$1,000 invested under compound interest at an annual effective interest rate of 5%?

Problem 2.4. At time 0, Roger deposits \$4,000 into an account which earns an effective annual interest rate of 5%. At time 3 he makes another deposit in the amount of \$3,000. Those are the only activities in Roger's account.
Harry's account earns an annual effective interest rate of 12%. Harry makes a single deposit of \$7,000 at an unknown time $t^* \in [0, 5)$. Harry makes no withdrawals from his account.
Roger and Harry compare the balances in their accounts at time 5 and realize they are equal. Find t^*.

(a) About 3.
(b) About 3.4.
(c) About 3.7.
(d) About 4.
(e) None of the above.