d.... simple discount rate

$\Rightarrow$ the discount f'tion is linear:

$$v(t) = 1 - d\cdot t \qquad \text{for } t < \frac{1}{d}$$

$\Rightarrow$ the accumulation f'tion is of the form:

$$\boxed{a(t) = \frac{1}{v(t)}} \underset{\text{simple discount}}{=} \frac{1}{1-d\cdot t}$$

TRUE IN GENERAL

$\Rightarrow$ for the effective discount rate in year 5:

$$d_5 = \frac{a(5)-a(4)}{a(5)} = \dots$$

M329F: January 29th, 2020.

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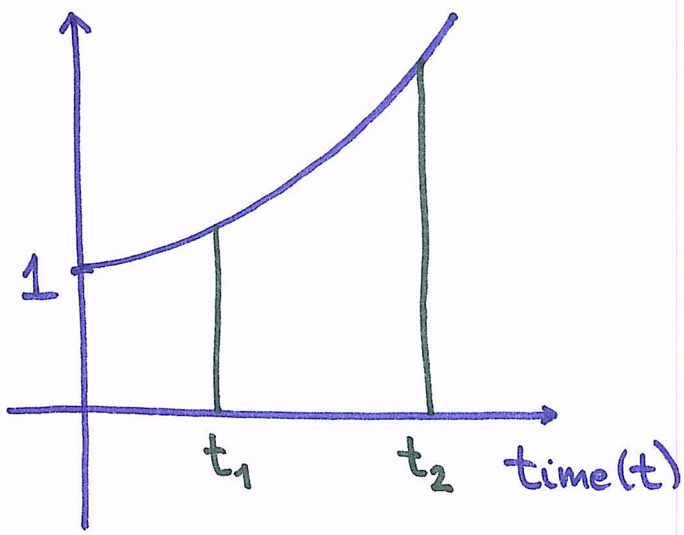
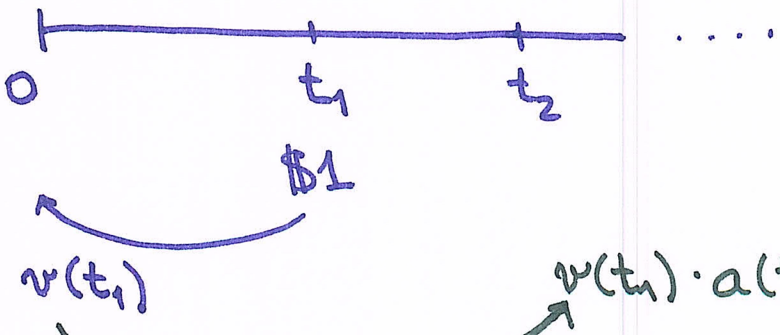
EQUIVALENCE OF EFFECTIVE INTEREST AND DISCOUNT RATES OVER $[t_1, t_2]$

Account I: accumulation f'tion $a(\cdot)$



Q: If one invests \$1 @ time t_1, what is the balance @ time t_2 (in terms of the accumulation f'tion)?



$$v(t_1) \cdot a(t_2) = \frac{a(t_2)}{a(t_1)}$$

On the other hand,

$$i_{[t_1,t_2]} = \frac{a(t_2)-a(t_1)}{a(t_1)} = \frac{a(t_2)}{a(t_1)} - 1$$

The \$1 invested @ time $\cdot t_1$ grows to

$$\boxed{\frac{a(t_2)}{a(t_1)} = 1 + i_{[t_1,t_2]}} \quad \text{until time } t_2$$

<u>Account II</u>. accumulation f'tion $\tilde{a}(\cdot)$

The \$1 invested @ time $\cdot t_1$ grow to

$$\frac{\tilde{a}(t_2)}{\tilde{a}(t_1)} \quad \text{until time} \cdot t_2$$

On the other hand, by def'n,

$$d_{[t_1,t_2]} = \frac{\tilde{a}(t_2)-\tilde{a}(t_1)}{\tilde{a}(t_2)} = 1 - \frac{\tilde{a}(t_1)}{\tilde{a}(t_2)}$$

$$\Rightarrow \quad \frac{\tilde{a}(t_1)}{\tilde{a}(t_2)} = 1 - d_{[t_1,t_2]}$$

$\Rightarrow$ The \$1 invested @ time $\cdot t_1$ grows to

$$\boxed{\frac{\tilde{a}(t_2)}{\tilde{a}(t_1)} = \frac{1}{1-d_{[t_1,t_2]}}} = \left(1-d_{[t_1,t_2]}\right)^{-1}$$

$\Rightarrow$ In order for the accounts to be equivalent over $[t_1, t_2]$, we must have

$$\boxed{\begin{gathered} 1 + i_{[t_1,t_2]} = \frac{1}{1 - d_{[t_1,t_2]}} \\ \Longleftrightarrow \\ (1+i_{[t_1,t_2]})(1-d_{[t_1,t_2]}) = 1 \end{gathered}}$$