

**Problem 3.1.** (5 points) You are given the following table of spot rates:

Length of Investment	Spot rate
1 year	0.04
2 years	0.045
3 years	0.05

Find the implied forward rate for the time period  $[1, 3]$ .

- (a) 0.045
- (b) 0.055
- (c) 0.09
- (d) 0.11
- (e) None of the above.

**Problem 3.2.** (5 points) *Source: Example 7.1.1 from the textbook.*

At the beginning of a quarter, Roger's aunt Agatha buys 100 shares of preferred stock  $S$  at a price to provide her a yield of 5.5% assuming that all the dividends are paid. The stock  $S$  is nonadjustable and nonparticipating which implies that the dividend schedule which was in force at the time of purchase cannot be altered. So, the dividends are fixed at the times and levels that are stipulated at issuance, namely \$0.36 per share each quarter.

What is the fair price that aunt Agatha should pay?

- (a) 654.545
- (b) 1239.678
- (c) 1671.38
- (d) 2671.58
- (e) None of the above.

**Problem 3.3.** (5 points) Roger borrows 1000 for 10 years at an annual effective interest rate of 10%. He can choose to repay this loan using the amortization method with equal payments  $P$  paid at the end of each year.

Find  $P$ .

- (a) About 113.7
- (b) About 162.7
- (c) About 213.7
- (d) About 262.7
- (e) None of the above

**Problem 3.4.** (5 points) Roger borrows 1000 for 10 years at an annual effective interest rate of 10%. He can choose to repay this loan using the amortization method with equal payments  $P$  paid at the end of each year.

Instead, Roger chooses to repay the loan using a **sinking fund** that pays an annual effective rate of 14%.

He will still be paying the same amount  $P$  at the end of every year. However, the amount  $P$  first goes towards the repayment of the interest of the loan and the rest is deposited in the sinking fund.

Determine the balance in the sinking fund at the end of the 10 years after the loan is repaid.

- (a) About 113
- (b) About 163
- (c) About 213
- (d) About 263
- (e) None of the above

**Problem 3.5.** (5 points) Consider two bonds with semiannual coupons both redeemable at par.

The first bond is a 5%-bond with face amount 1000 and it is redeemable in  $k$  years. Its price at issue is 1300.

The second bond is a 4%-bond with the same face amount of 1000 and it is also redeemable in  $k$  years. Its price at issue is 1100.

The nominal yield rate for the two bonds is the same. Find this rate.

- (a) 0.035
- (b) 0.0375
- (c) 0.04
- (d) 0.0425
- (e) None of the above.

**Problem 3.6.** (5 points) You buy an  $n$ -year \$1,000 par value bond with 6.5% annual coupons assuming an annual yield rate of 9.25%. After the first two years, the bond's book value has changed by 23.76. Find the original purchase price of this bond.

- (a) 825.46
- (b) 865.82
- (c) 902.34
- (d) 925.74
- (e) None of the above.

**Problem 3.7.** (5 points) A \$200,000-par value 10-year bond with 10% annual coupons is bought at a premium to yield 8% for the first 5 years and 6% thereafter.

Calculate the interest portion of the 9<sup>th</sup> coupon.

- (a) About 10,880
- (b) About 11,680
- (c) About 12,880
- (d) About 14,810
- (e) None of the above.

**Problem 3.8.** (5 points) Roger purchased a 20-year par value bond with semiannual coupons at a nominal annual rate of 8% convertible semiannually. The price was 1722.25.

The bond can be **called** at par value  $X$  on any coupon date starting at the end of year 15 and after that coupon is paid. We do not know the value of  $X$ , but we know the following requirement of the bond purchase:

The price that Roger paid guarantees that he will **at least** receive a nominal annual interest rate of 6% convertible semiannually from this investment.

Calculate  $X$ .

- (a) 1440
- (b) 1540
- (c) 1580
- (d) 1640
- (e) None of the above.

**Problem 3.9.** (5 points) A perpetuity is payable continuously at the annual rate of  $1 + t^2$  at time  $t$ . Assume that the force of interest is constant at  $\delta = 0.05$ . Find the present value of the perpetuity.

- (a) 14,210
- (b) 15,080
- (c) 15,720
- (d) 16,020
- (e) None of the above.

**Problem 3.10.** (5 points) Consider the following bond which is supposed to account for inflation.

The bond's term is 8 years and the face value is \$1000. The redemption amount is  $1000 \cdot (1.025)^8$ , the first annual coupon is \$80 and each subsequent one is by 2.5% greater than the one preceding it.

Roger wishes to earn a non-inflation adjusted rate of 8% on this bond. How much should he be willing to pay?

- (a) 1000
- (b) 1025
- (c) 1155.33
- (d) 1175.74
- (e) None of the above.

**Problem 3.11.** (5 points) Consider a fund which earns an annual effective interest rate of 6% and with an initial balance of \$1,000. At the end of each year the interest accumulated during that year plus an additional \$100 is withdrawn.

Assume these amounts were deposited to a different account immediately after they were withdrawn from the above account. Let the second account earn an annual effective interest rate of 9%. Find the amount in the second account immediately after the last deposit to it is made.

- (a) About 1085
- (b) About 2015
- (c) About 2055
- (d) About 2085
- (e) None of the above.

**Problem 3.12.** (5 points) The following spot rates, in our usual notation, are observed

$$r_1 = 0.05, \quad r_2 = 0.055, \quad r_3 = 0.06, \quad r_4 = 0.065.$$

Calculate the level swap interest rate  $R$  on a loan with annual end-of-year payments. Assume that the payments are interest-only.

- (a) About 4.52%
- (b) About 6%
- (c) About 6.42%
- (d) About 7%
- (e) None of the above.