Notes: This is a closed book and closed notes exam. The maximal score on this exam is 50 points.
Time: 50 minutes

Problem 1.1. (5 pts) On January 1st of each odd-numbered calendar year, from 2023 through 2035, you expect to receive a payment of \$1,000. The effective annual interest rate is 12%.
  Find the present value P of this stream of payments on November 1st, 2021.

  (a) $P \leq 2,550$
  (b) $2,550 < P \leq 3,550$
  (c) $3,550 < P \leq 3,850$
  (d) $3,850 < P \leq 5,550$
  (e) None of the above

Problem 1.2. (5 points)*Source: Exam FM, May 2005, Problem #8.*
A loan is being repaid with 25 end-of-year payments of 300 each. The effective annual rate of interest charged on this loan is 8%. What is the outstanding loan balance right after the 10th payment?

Problem 1.3. (5 pts) *Source: Sample FM Exam Problems, Problem #2.*
Kathryn deposits \$100 into an account at the beginning of each $4-$year period for 40 years. The account credits interest at an unknown annual effective interest rate of i.
The accumulated amount in the account at the end of 40 years is X. The accumulated amount in the account at the end of 20 years is equal to Y.
  You are given that $X = 5Y$. Calculate X.

  (a) 4695
  (b) 5070
  (c) 5445
  (d) 6195
  (e) None of the above

Problem 1.4. (5 pts) *Source: FM Exam, May 2005, Problem #21.*
A discount electronics store advertises the following financing arrangement:

"We don't offer you confusing interest rates. We'll just divide your total cost by 10 and you can pay us that amount each month for a year."

The first payment is due immediately on the date of sale and the remaining eleven payments at monthly intervals thereafter. Calculate the effective **annual** interest rate i the customers are actually paying on their loans.

(a) 0.351

(b) 0.413

(c) 0.42

(d) 0.512

(e) None of the above

Problem 1.5. (5 pts) Roger makes an inital deposit of K into an account governed by the time-varying force of interest $\delta_t = \frac{9}{10}\sqrt{t}$ (per annum).

At the same time, Harry makes an initial deposit at the same amount into an account governed by the constant annual discount rate d.

There are no subsequent deposits to or withdrawals from either of the two accounts.

After 4 years, Roger and Harry realize that the balances in their accounts are equal. Which of the following is the closest to d?

(a) $e^{-6/5}$

(b) $e^{-1/5}$

(c) $1 - e^{-1/5}$

(d) $1 - e^{-6/5}$

(e) 1

Problem 1.6. (5 pts) A fund had a balance of 75 on January 1st and 60 on December 31st of the same year. At the end of every month during that year, there was a deposit of 10 onto the fund's account.

There were altogether four withdrawals during that same year:

(1) 5 on February 28th;
(2) 25 on June 30th;
(3) 80 on October 15th;
(4) 35 on October 31st (presumably, for candy).

Find the dollar-weighted rate of return for this year using the **simple interest** approximation.

(a) About 9.0%

(b) About 9.5%

(c) About 10.0%

(d) About 11.0%

(e) None of the above

Problem 1.7. (5 pts) Find the accumulated value of a $10-$year annuity-immediate of $100 per year if the first six payments are invested at an effective rate of interest of 5% and the last four payments are invested at an effective rate of interest of 4%.

Be careful: The interest rate switches regimes at time 6.

(a) About $1,150$

(b) About $1,200$

(c) About $1,220$

(d) About $1,350$

(e) None of the above

Problem 1.8. (5 pts) Assume that an annuity-immediate provides 10 annual payments. The first payment is equal to $100 and the payments increase in such a way that each subsequent payment is 4% greater than the one preceding it. If the annual effective interest rate equals 7%, find the present value of this annuity.

(a) 825

(b) 833

(c) 843

(d) 853

(e) None of the above

Problem 1.9. (5 points) A company deposits $1,000 at the beginning of the first year and $150 at the beginning of each subsequent year into perpetuity.

In return, the company receives payments at the end of every year forever. The first payment equals $100 and each subsequent increases by 5%.

Calculate the company's yield rate for this investment (under the condition that the yield rate exceeds 5%).

    (a) 0.047

    (b) 0.057

    (c) 0.067

    (d) 0.077

    (e) 0.087

Problem 1.10. (5 pts) Roger makes an initial investment of $100,000.

In return, he gets cash flows of $40,000 at the end of each of years 2, 3 and 4. The cash flows can be reinvested at 4.0% per annum effective.

If the rate of interest at which the investment is to be valued is 5.0%, let P denote the net present value of this investment today. Then,

    (a) $2,725.92$

    (b) $4,567.82$

    (c) $12,784.32$

    (d) $102,567.24$

    (e) None of the above.

Problem 1.11. Consider an annuity immediate with the following regime of payments:

- end-of year payments equal to 10 for 7 years;
- end-of year payments equal to 5 for the following 5 years;
- end-of year payments equal to 8 for the following 3 years;
- end-of year payments equal to 6 for the following 5 years.

Let the present value of the above annuity immediate be denoted by P. In standard actuarial notation, you are given that

$$a_{\overline{7}|} = 5.7864, \quad a_{\overline{12}|} = 8.8633, \quad a_{\overline{15}|} = 10.378, \quad \text{and} \quad a_{\overline{20}|} = 12.4622.$$

Find P.

(a) 91.32

(b) 97.87

(c) 86.32

(d) 107.34

(e) 102.67

Problem 1.12. On January 1, 1997, Brian's stock portfolio is worth $100,000. On September 30, 1997, $5,000 is withdrawn from the portfolio, and immediately after this withdrawal the portfolio has a value of $105,000. Twelve months later, the value of the portfolio is $108,000 and Brian adds $3,000 worth of stock to his portfolio. On December 31, 1998, the portfolio is worth $100,000. What is the effective time-weighted rate of return for Brian's stock portfolio over the two-year period?
Be careful: The question is asking you for the effective two-year interest rate.

(a) 0.0193

(b) 0.02913

(c) 0.0257

(d) 0.00456

(e) 0.0374