University of Texas at Austin

The time-varying force of interest.

Please, provide your final answer only to the following questions:

Problem 2.1. (2 pts) Let

$$a(t) = (1 + 0.05)^{3t} (1 + 0.02)^{t/2}$$
.

The force of interest associated with the above accumulation function is constant. True or false?

Problem 2.2. (2 points) In our usual notation, with δ_t denoting the time-varying force of interest, the discount function can be written as

$$v(t) = e^{-\int_0^t \delta_u \, du}.$$

True or false?

Problem 2.3. (2 points) Let the accumulation function be of the form

$$a(t) = \alpha t^2 + \beta t + \gamma.$$

Let a(1) = 1.12 and a(2) = 1.44. Then, the force of interest at time t = 1.5 is less than 0.255. True or false?

Problem 2.4. (2 points) Let the force of interest be given by $\delta_t = \frac{1}{1+t}$. Then, the accumulation function is of the form a(t) = 1 + t. True or false?

Problem 2.5. (2 points) Let the force of interest be given by $\delta_t = \frac{2}{1+t}$. Then, the accumulation function is of the form $a(t) = 1 + t^2$. True or false?

Problem 2.6. (5 pts) Roger makes an inital deposit of K into an account governed by the time-varying force of interest $\delta_t = \frac{9}{10}\sqrt{t}$ (per annum).

At the same time, Harry makes an initial deposit at the same amount into an account governed by the constant annual discount rate d.

There are no subsequent deposits to or withdrawals from either of the two accounts.

After 4 years, Roger and Harry realize that the balances in their accounts are equal. Which of the following is the closest to d?

- (a) $e^{-6/5}$
- (b) $e^{-1/5}$
- (c) $1 e^{-1/5}$
- (d) $1 e^{-6/5}$
- (e) 1