M329F Theory of Interest
University of Texas at Austin
In-Term Exam I
Instructor: Milica Čudina

Notes: This is a closed book and closed notes exam. The maximal score on this exam is 100 points.
Time: 75 minutes

TRUE/FALSE

1 (2)	TRUE	FALSE
2 (2)	TRUE	FALSE
3 (2)	TRUE	FALSE
4 (2)	TRUE	FALSE

MULTIPLE CHOICE

1 (3)	a	b	c	d	e
2 (3)	a	b	c	d	e

FOR GRADER'S USE ONLY:

T/F	1.	2.	3.	4.	5.	6.	7.	M.C.	Σ

1.1. **DEFINITIONS.**

Problem 1.1. (5 points) Provide the definition of an *effective interest rate for the time period* $[t_1, t_2]$ in terms of the accumulation function $a(\cdot)$.

Problem 1.2. (5 points) Provide the definition of the *force of interest* in terms of the accumulation function $a(\cdot)$ in the compound interest case.

1.2. **TRUE/FALSE QUESTIONS.** *Please, note your answers on the front page.*

Problem 1.3. (2 pts) In our usual notation, for equivalent $d^{(n)}$ and $i^{(p)}$, we have that $d^{(n)} > i^{(p)}$. *True or false?*

Problem 1.4. (2 pts) The constant nominal rates of interest may be converted to an equivalent constant force of interest. *True or false?*

Problem 1.5. (2 pts) The discount function $v(t)$ is in most cases increasing. *True or false?*

Problem 1.6. (2 pts) The effective rate of interest measures the amount of interest earned over the period relative to the balance at the beginning of the period. *True or false?*

Problem 1.7. (2 pts) In the usual notation, the following equality is true:

$$d = iv$$

True or false?

Problem 1.8. (2 pts) If interest rate is credited to the initial investment amount only, we say that simple interest is being used. *True or false?*

Problem 1.9. (2 pts) Assume that the interest rate is strictly positive. If a future payment is made sooner than what was originally predicted, then its present value decreases. *True or false?*

Problem 1.10. (2 points) You are given that, in our usual notation,

$$A_K(t) = \frac{1,000}{100 - t} \quad 0 \leq t < 100.$$

Then, $K = 1000$. *True or false?*

1.3. **Free-response problems.** Please, explain carefully all your statements and assumptions. Numerical results or single-word answers without an explanation (even if they're correct) are worth 0 points.

Problem 1.11. (10 points) Find the accumulated value of \$5,000 at the end of 30 years and 4 months at 6% per annum convertible semiannually:

(1) (5 pts) assuming compound interest throughout;
(2) (5 pts) assuming simple interest during the final fractional period.

Problem 1.12. (9 points) Roger invests \$1000 into an account whose growth is governed by the accumulation function

$$a(t) = \alpha t^2 + 4\beta, \text{for every } t \geq 0$$

where α and β are unknown constants.

(i) (3 pts) Find the value of the parameter β.

(ii) (3 pts) Additionally, assume that the value of Roger's investment at time 4 equals \$1250. Find the value of the parameter α.

(iii) (3 pts) Find the accumulated value of Roger's investment at time 10.

Problem 1.13. (10 points) Roger wins an actuarial lottery which promises to pay him \$3,000 at the end of each of the following three years.

Alternatively, he is offered the "Instant Cash Option" by which he would immediately get the net present value of the original payments calculated with respect to the nominal annual interest rate of 9% payable quarterly.

(i) (5 pts) What is the amount that Roger would receive if he opts for the "Instant Cash Option"?

(ii) (5 pts) Assume that Roger decides to take the NPV of the three payments and invests that sum into an account at an effective rate of discount of 7.25%. What is the amount that Roger has in that account after 4 years?

Problem 1.14. (6 points) Assume our usual notation. The accumulation of interest in an account is governed by the following:

(1) a nominal interest rate convertible semiannually of 9% for the first 3 years,
(2) a nominal discount rate convertible quarterly of 10% for the following 2 years,
(3) $d = 7\%$ for the next 2 years,
(4) $i = 12\%$ thereafter.

Roger makes an initial deposit of \$100. He makes no subsequent deposits or withdrawals, and liqudates his account after 10 years. Calculate the amount he is able to withdraw at that time.

1.4. **MULTIPLE CHOICE QUESTIONS.** *Please, note your answers on the front page.*

Problem 1.15. (5 pts) An investor invests a certain amount of money at time 0 into an account earning 5.75% simple interest. How many years will it take to double her money (round your answer to the nearest integer)?

(a) About 2 years
(b) About 5 years
(c) About 11 years
(d) About 17 years
(e) None of the above

Problem 1.16. (5 pts) An investor invests a certain amount of money at time 0 into an account earning 5% compound interest. How many years will it take to double her money (round your answer to the nearest integer)?

(a) About 2.5 years
(b) About 12.5 years
(c) About 22.5 years
(d) About 32.5 years
(e) None of the above

Problem 1.17. (5 pts) An investor invests \$1,000 at time 0 into an account earning 6.0% annual simple interest. What is the effective interest in the fifth year of the investment (round your answer to two decimal places)?

(a) 4.84%
(b) 5.95%
(c) 6.16%
(d) 7.27%
(e) None of the above

Problem 1.18. (5 pts) Let the accumulation function for an investment account be given by

$$a(t) = 1 + 3z \cdot t \,\text{for every } t \geq 0$$

One hundred dollars invested in the account at time 0 grows to \$420 at time 10. Find the value of z.

(a) 22/3
(b) 32/3
(c) 52/3
(d) 62/3
(e) None of the above

Problem 1.19. (5 pts) To save for a car, Roger opens a bank account into which he initially deposits \$3,200. He deposits $\$3,500$ into the same account six months later, and \$800 three months after that. In our usual notation, assume that $i^{(2)} = 0.08$. Which is the price of the most expensive car Roger can buy at time 1 using only the money from her account?

(a) About $\$7,509$
(b) About $\$7,798$
(c) About $\$7,859$
(d) About $\$7,917$
(e) None of the above

Problem 1.20. (5 points) *Source: Course 2 Exam, May 2003, Problem #1.*
Bruce deposits 100 into a bank account. His account is credited interest at a nominal rate of interest $i^{(2)}$ convertible semiannually.
At the same time, Peter deposits 100 into a separate account. Peter's account is credited interest at a force of interest of δ.
After 7.25 years, the value of each account is 200. Calculate $|i^{(2)} - \delta|$.

(a) 0.12%
(b) 0.23%
(c) 0.31%
(d) 0.39%
(e) None of the above.

Problem 1.21. (5 points) Roger makes an initial investment of \$100,000.

In return, he gets cash flows of \$40,000 at the end of each of years 2, 3 and 4. The cash flows can be reinvested at 4.0% per annum effective.

If the rate of interest at which the investment is to be valued is 5.0%, let P denote the net present value of this investment today. Then,

(a) $P < 0$
(b) $0 \leq P < 700$
(c) $700 \leq P < 1400$
(d) $1400 \leq P < 1600$
(e) None of the above.

Problem 1.22. *Source: SoA Exam FM/2, November 1992, Problem #1.*
Given a nominal interest rate of 7.5% convertible semiannually, determine the sum of the equivalent

i. force of interest, and
ii. nominal discount rate convertible quarterly.

(a) 0.1460
(b) 0.1463
(c) 0.1466
(d) 0.1469
(e) None of the above.