M329F *Theory of Interest*
Spring 2020
University of Texas at Austin
Sample In-Term Exam II
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Notes: This is a closed book and closed notes exam. The maximal score on this exam is 50 points.
Time: 50 minutes

1.1. **TRUE/FALSE QUESTIONS.**

Problem 1.1. (2 pts) In our usual notation, we have the following:

$$\frac{\ddot{a}_{\overline{n}|\,i}}{a_{\overline{n}|\,i}} = v.$$

True or false?

Problem 1.2. (2 pts) Let

$$a(t) = (1+0.05)^{3t}(1+0.02)^{t/2}\,.$$

The force of interest associated with the above accumulation function is constant. *True or false?*

Problem 1.3. (2 pts) Consider a level perpetuity-immediate with the fair price P purchased to yield the effective rate of y per payment period. Then, every payment of the perpetuity is exactly equal to the interest earned on P. *True or false?*

Problem 1.4. (2 pts) Consider a 20−year annuity with every payment equal to 100 and the first payment in two years from today. Then, the present value of this annuity can be expressed as

$$100v^3\ddot{a}_{\overline{20}|\,i}$$

where i denotes the effective annual interest rate and v is the discount factor associated with it. *True or false?*

Problem 1.5. (2 pts) You make annual deposits equal to \$200 with the first payment being today. There are no other withdrawals or deposits. Then, the balance in your account 12 years from now can be expressed as

$$200(1+i)^3 s_{\overline{10}|\,i}$$

where i denotes the effective annual interest rate. *True or false?*

1.2. **FREE-RESPONSE PROBLEMS..** Please, explain carefully all your statements and assumptions. Numerical results or single-word answers without an explanation (even if they're correct) are worth 0 points.

Problem 1.6. (5 points) You make an initial investment of \$1,000 and an additional investment of \$1,000 at time 1.
The balance at time 1, just before the deposit is made equals \$1,200, while the final balance at time 2 equals \$2,200.

Find the (approximate) dollar-weighted rate of return per annum.

Problem 1.7. (8 pts) *Note: Compare to Problem 2.5.1 from the textbook.*
Roger borrows \$5,000 from Sally and agrees to repay the debt in two installments \$2,500 in two years and another \$3,500 in another 2 years (i.e., 4 years after the initial loan is made).

(i) (3 pts) Find the interest rate i that Roger is being charged on the above loan.

(ii) (5 pts) Right after Roger repays the first installment at time 2, Sally reinvests that money at the effective annual interest rate $j = 5\%$. She liquidates that account at time 4 and is interested how well she did with the entire investment.
Find Sally's yield rate y.

1.3. MULTIPLE CHOICE QUESTIONS.

Problem 1.8. (5 points) Roger wants to be able to buy a perpetuity-immediate in exactly 3 years. This perpetuity should make payments equal to \$1,000 at the end of every quarter.

Assume that the annual effective interest rate equals $i = 9.5\%$.

Let P denote the fair price of the perpetuity-immediate that Roger wants to purchase (of course, valued at the time of purchase!). Then,

(a) $P \leq 21,000$

(b) $21,000 < P \leq 41,000$

(c) $41,000 < P \leq 61,000$

(d) $61,000 < P \leq 81,000$

(e) None of the above

Problem 1.9. (5 points) A 5-year annuity-immediate pays 100 the first year and each subsequent payment is 2% smaller than the one preceding it. Let the present value of this annuity if the effective annual interest rate equals $i = 3\%$ be denoted by PV. Then,

(a) $PV \leq 210$

(b) $210 < PV \leq 410$

(c) $410 < PV \leq 610$

(d) $610 < PV \leq 810$

(e) None of the above

Problem 1.10. (5 points) Find $\ddot{a}_{\overline{10}|}$ assuming compound interest with the effective interest rate per payment period of 0.07

(a) $\ddot{a}_{\overline{10}|} < 5.41$

(b) $5.41 \leq \ddot{a}_{\overline{10}|} < 6.39$

(c) $6.39 \leq \ddot{a}_{\overline{10}|} < 7.38$

(d) $7.38 \leq \ddot{a}_{\overline{10}|} < 8.37$

(e) None of the above

Problem 1.11. (5 pts) Roger wants to accumulate \$250,000 on a savings account that earns 12% interest, convertible quarterly. He is ready to make deposits equal to \$1,000 at the beginning of every month.

How long (in months, rounded to the nearest month) will it take to accumulate the desired balance?

(a) 71
(b) 91
(c) 101
(d) 111
(e) None of the above

Problem 1.12. (5 pts) As part of a legal settlement an injured person is awarded a series of payments as compensation for her medical costs. The injured person is entitled to 25 annual payments. The first payment happens immediately and equals \$20,000 now. To account for the projected inflation rate in the medical fields, each subsequent annual payment will be 3% higher than the previous year's payment.

Assume that the effective annual interest rate equals 10%. Let the total present value of all the payments under this interest rate be denoted by S. Then,

(a) $S \approx 183,550$
(b) $S \approx 233,550$
(c) $S \approx 253,550$
(d) $S \approx 258,550$
(e) None of the above

Problem 1.13. (5 pts) On January 1^{st} of each odd-numbered calendar year, from 2023 through 2035, you expect to receive a payment of \$1,000. The effective annual interest rate is 12%.

Find the present value P of this stream of payments on November 1^{st}, 2021.

(a) $P \leq 2,550$
(b) $2,550 < P \leq 3,550$
(c) $3,550 < P \leq 3,850$
(d) $3,850 < P \leq 5,550$
(e) None of the above

Problem 1.14. (5 points) *Source: SoA, May 1988, Problem #1.*
You are given that $\delta_t = \frac{2}{t-1}$ for $2 \leq t \leq 10$. For any one-year interval between n and $n+1$ with $2 \leq n \leq 9$, calculate the equivalent $d^{(2)}$.

(a) $\frac{1}{n}$
(b) $\frac{2}{n}$
(c) $\frac{n-1}{n}$
(d) $\frac{n}{n-1}$
(e) $(\frac{n}{n-1})^2$

Problem 1.15. (5 points)*Source: CAS, May 1991, Problem #4.*
Jim borrows \$5,000 from a bank now, an additional \$3,000 one year from now, and an additional \$2,000 five years from now. At what time t^* (in years) would a single payment of \$10,000 be equivalent at a nominal rate of interest of 12% convertible monthly?

(a) $0 \leq t^* < 0.9$
(b) $0.9 \leq t^* < 1$
(c) $1 \leq t^* < 1.1$
(d) $1.1 \leq t^* < 1.2$
(e) $1.2 \leq t^* < \infty$

Problem 1.16. Bertie invests $4,000 today and in return he gets:

- $1,000 in one year,
- $2,000 in two years, and
- $3,000 in three years.

What is the annual effective yield rate on Bertie's investment?

(a) 0.0456

(b) 0.1077

(c) 0.1944

(d) 0.2733

(e) None of the above.

Problem 1.17. *Source: SoA, May 1998, Problem #9.*
On January 1, 1999 Luciano deposits 90 into an investment account. On April 1, 1999, when the amount in Luciano's account is equal to X, a withdrawal of W is made. No further deposits or withdrawals are made to Luciano's account for the remainder of the year. On December 31, 1999, the amount in Luciano's account is 85. The dollar-weighted return over the one-year period is 20%. The time-weighted return over the one-year period is 16%. Calculate X.

(a) 101.63

(b) 103.63

(c) 105.63

(d) 107.63

(e) 109.63

Problem 1.18. *Source: An old Sample SoA problem set.*
Susan and Jeff each make deposits of 100 at the end of each year for 40 years. Starting at the end of the 41^{st} year, Susan makes annual withdrawals of X for 15 years and Jeff makes annual withdrawals of Y for 15 years. Both funds have a balance of 0 after the last withdrawal.

Susan's fund earns an annual effective interest rate of 8% while Jeff's fund earns an annual effective interest rate of 10%. Calculate $Y - X$.

(a) 2792

(b) 2824

(c) 2859

(d) 2893

(e) 2925

Problem 1.19. (5 points) Consider an annuity immediate with the following regime of payments:

- end-of year payments equal to 5 for 7 years;
- end-of year payments equal to 15 for the following 5 years;
- end-of year payments equal to 10 for the following 3 years;
- end-of year payments equal to 6 for the following 5 years.

Let the present value of the above annuity immediate be denoted by P. In standard actuarial notation, you are given that

$$a_{\overline{7}|} = 5.7864, \quad a_{\overline{12}|} = 8.8633, \quad a_{\overline{15}|} = 10.378, \quad \text{and} \quad a_{\overline{20}|} = 12.4622.$$

Find X.

(a) 101.74

(b) 102.74

(c) 103.64

(d) 105.84

(e) None of the above.

Problem 1.20. (5 points) *Source: CAS, May 1999, Problem #9.*
Peter borrows \$5,000 from Kevin for a term of five years. Peter agrees to pay interest at the end of each year at an annual effective interest rate of 8% and to repay the entire \$5,000 as a lump sum at the end of five years. Immediately after the third payment, Kevin sells his right to future payments to Martha at a price that will yield Martha an effective annual rate of 5%. Let Kevin's overall effective annual yield rate be denoted by y. Then, we can say that

(a) $y < 0.086$

(b) $0.086 \leq y < 0.09$

(c) $0.09 \leq y < 0.094$

(d) $0.094 \leq y < 0.098$

(e) $0.098 \leq y$