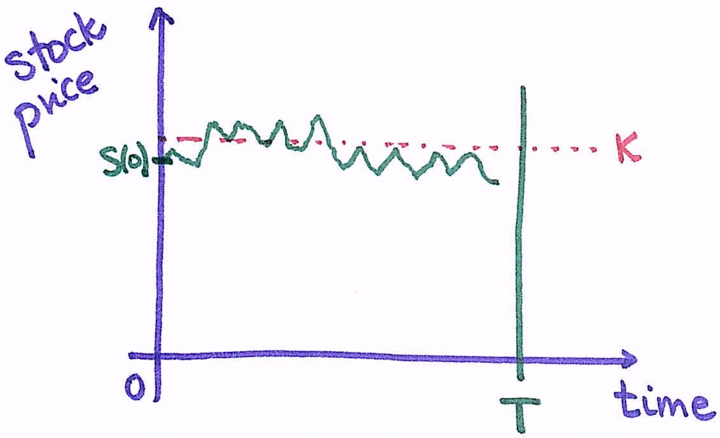
Asian Options.

04/23/2018.

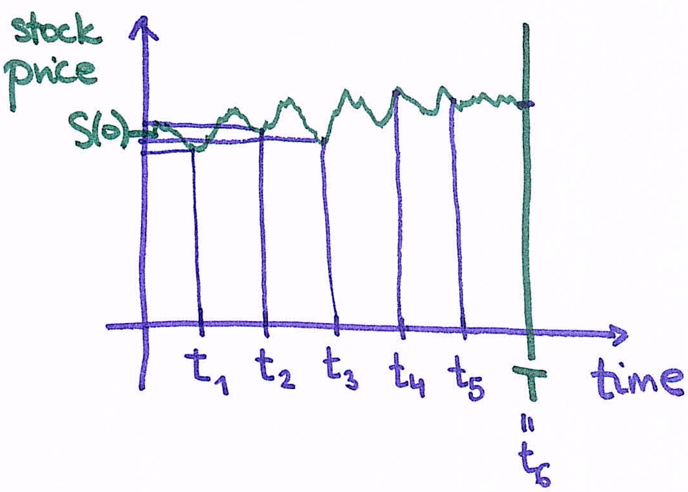
* A type of EXOTIC OPTIONS : see chooser, gap,... *

Look @ a regular call: Its payoff is $V_c(T) = (S(T) - K)_+$
Imagine that you own this call.
Look @ the evolution of the stock price during the life of the option.



A large investor could manipulate the stock price @ end of the time horizon to affect the payoff of the option.

Idea: construct an option which has an average of stock prices in its payoff.
(sampled during the life of the option)



$$\left(S(t_k),\ k = 1..n\right) \longmapsto \text{average stock price}$$

average?

Set: $\begin{cases} A(T) = \frac{1}{n}\left(S(t_1) + \dots + S(t_n)\right) & \text{arithmetic average} \\ G(T) = \left(S(t_1) \cdot \ldots \cdot S(t_n)\right)^{1/n} & \text{geometric average} \end{cases}$

Fact: $\boxed{A(T) \geq G(T)}$

The basis of the payof — call / put

and

the stock price average can be used as — the underlying / the strike price.

PAYOFFS

	Geometric		Arithmetic
Strike Price	Call: $(S(T)-G(T))_+$	$\geq$	$(S(T)-A(T))_+$
	Put: $(G(T)-S(T))_+$	$\leq$	$(A(T)-S(T))_+$
Underlying (w/ strike K given)	Call: $(G(T)-K)_+$	$\leq$	$(A(T)-K)_+$
	Put: $(K-G(T))_+$	$\geq$	$(K-A(T))_+$

$\Downarrow$

Analogous inequalities hold for option prices. :)

Problem. No dividends; $S(0) = 100$; $\sigma = 0.30$.

$r = 0.04$

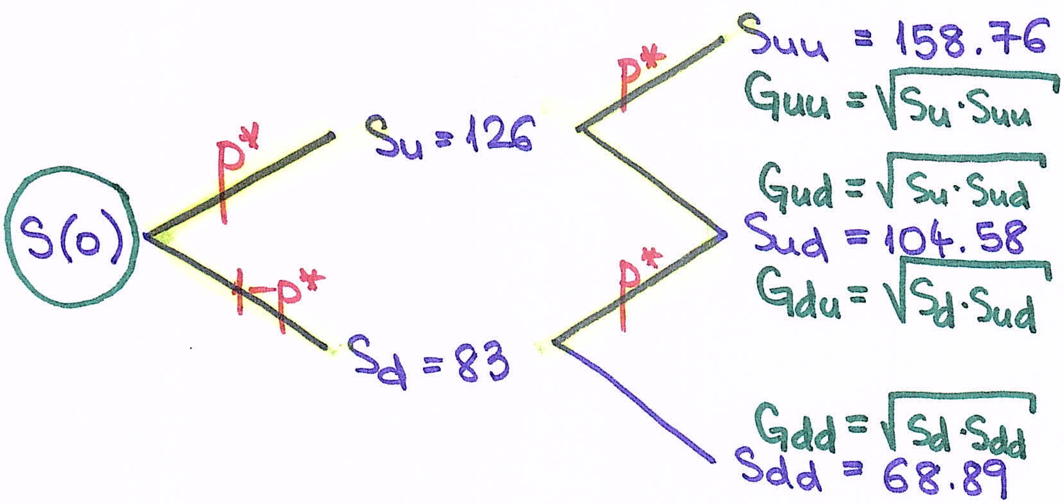
We model the stock·price evolution over the next year using a two·period FORWARD binomial tree.

Consider an Asian, geometric·average·strike <u>call</u> w/ exercise date in one year. Price this option using the above tree!

$\delta = 0; h = 1/2$

$\rightarrow$: $u = e^{(r-\delta)h + \sigma\sqrt{h}} \overset{\downarrow}{=} e^{0.04 \cdot 0.5 + 0.3\sqrt{0.5}} = 1.26$

$d = e^{(r-\delta)h - \sigma\sqrt{h}} = e^{0.02 - 0.3\sqrt{0.5}} = 0.825$



Payoff: $(S(T) - G(T))_+$

up·up: $V_{uu} = (S_{uu} - G_{uu})_+ = \left(S(0)\cdot u^2 - \sqrt{S(0)\cdot u \cdot S(0)\cdot u^2}\right)_+$

$= S(0)\cdot u \cdot (u - \sqrt{u})_+ = \cdots = \underline{17.33}$ w/ $(p^*)^2$

up·down: $V_{ud} = S(0)\cdot u(d - \sqrt{d})_+ = 0$

down·up: $V_{du} = S(0)\cdot d(u - \sqrt{u})_+ = \cdots = \underline{11.41}$ w/ $p^*(1-p^*)$

down·down: $V_{dd} = 0$

Our risk-neutral price: $V(0) = e^{-rT}\,\mathbb{E}^*[V(T)]$

$$p^* = \frac{1}{1+e^{\sigma\sqrt{h}}} = \frac{1}{1+e^{0.3\sqrt{0.5}}} = 0.447$$

$$\Rightarrow V(0) = e^{-0.04}\left[(0.447)^2 \cdot 17.33 + 0.447(1-0.447)\cdot 11.41\right] = 6.04 \;\blacksquare$$