Barrier Options [cont'd]

Example. [Rebate option]

Pmt of R @ time-T, if the barrier H was ever crossed/touched by the stock price during $[0,T]$.

- If $S(0) < H$, then rebate if $M(T) \geq H$

  w/ $M(T) := \max_{0 \leq t \leq T} S(t)$

  $\Rightarrow$ PAYOFF: $\boxed{V(T) = R \cdot \mathbb{I}_{[M(T) \geq H]}}$

- If $S(0) > H$, then rebate if $m(T) \leq H$

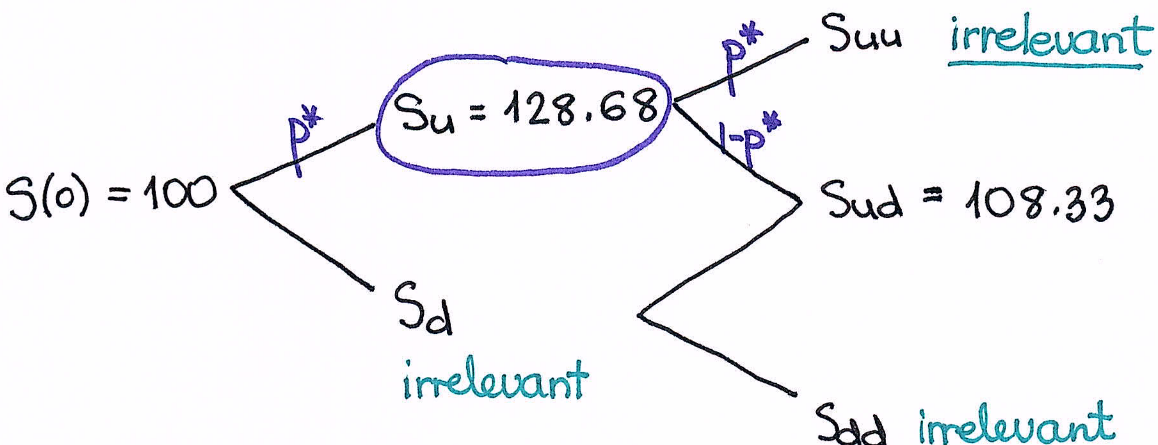
  w/ $m(T) := \min_{0 \leq t \leq T} S(t)$

  $\Rightarrow$ PAYOFF: $\boxed{V(T) = R \cdot \mathbb{I}_{[m(T) \leq H]}}$

Exercise. Consider a non-dividend-paying stock w/ $S(0) = 100$ and whose volatility is 0.3.
We use a TWO-PERIOD FORWARD binomial tree to model the stock price over the following year.
Assume that the continuously compounded, risk-free interest rate equals 0.08.

A rebate option pays \$20 @ time-1 if the stock price ever reaches the level \$110 during the following year.
What is the price of this rebate option?

A $\boxed{\text{FORWARD}}$ TREE:
$$u = e^{(r-\delta)h + \sigma\sqrt{h}} = e^{0.08\cdot\frac{1}{2} + 0.3\sqrt{\frac{1}{2}}} = 1.2868$$
$$d = e^{(r-\delta)h - \sigma\sqrt{h}} = e^{0.04 - 0.3\sqrt{\frac{1}{2}}} = 0.8419$$



$$p^* = \frac{1}{1+e^{\sigma\sqrt{h}}} = \frac{1}{1+e^{0.3\sqrt{\frac{1}{2}}}} = 0.447$$

PAYOFF: $\begin{cases} R & \text{w/ probab. } p^* \\ 0 & \text{w/ probab. } 1-p^* \end{cases}$

$$V_R(0) = e^{-0.08}\cdot 20 \cdot 0.447 \cong 8.26 \blacksquare$$

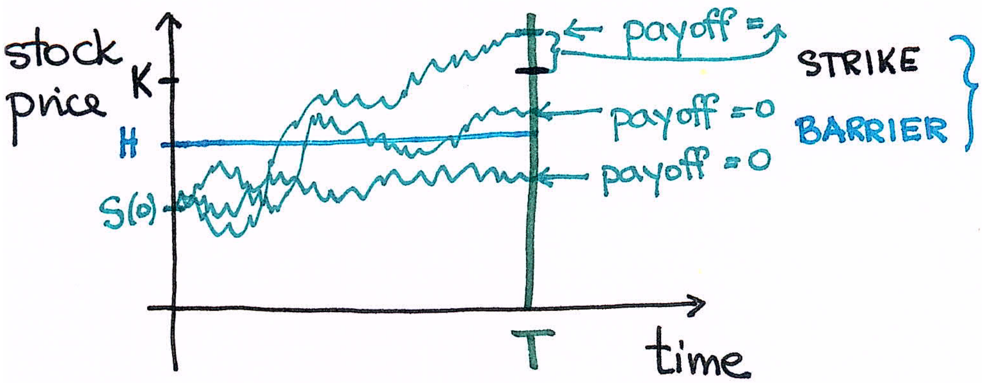
Family of ______ options:

_____ -and- _____ _____

up/down in/out call/put

$\Rightarrow$ 8 specific types

Example: An up-and-in call w/ strike K and barrier H. Say, $S(0) < H$.



A "degenerate" case: equivalent to a "vanilla" call w/ strike K.

In general: $V(T) = (S(T)-K)_+ \mathbb{I}_{[M(T) \geq H]}$

Example: An up-and-out put

PAYOFF: $V(T) = (K-S(T))_+ \mathbb{I}_{[M(T) < H]}$

Q: Consider an up-and-out call w/ $K > H$. What can you say about it? PAYOFF $\equiv 0$ $\Rightarrow$ PRICE $\equiv 0$.

Q: Consider the following portfolio:

$\begin{cases} \cdot \text{ up-and-in option} \\ \cdot \text{ up-and-out option} \end{cases}$ OTHERWISE IDENTICAL

PAYOFF: With the payoff of the vanilla version of the option $V(T)$:

$$V(T) \cdot \mathbb{I}_{[M(T) \geq H]} + V(T)\,\mathbb{I}_{[M(T) < H]} =$$

$$= V(T)\left[\mathbb{I}_{[M(T) \geq H]} + \mathbb{I}_{[M(T) < H]}\right] = V(T)$$

$\Rightarrow$ We had a replicating portfolio for the "regular" option.

2. You have observed the following monthly closing prices for stock XYZ:

Date	Stock Price
January 31, 2008	105
February 29, 2008	120
March 31, 2008	115
April 30, 2008	110
May 31, 2008	115
June 30, 2008	110
July 31, 2008	100
August 31, 2008	90
September 30, 2008	105
October 31, 2008	125
November 30, 2008	110
December 31, 2008	115

The following are one-year European options on stock XYZ. The options were issued on December 31, 2007.

(i) An arithmetic average Asian call option (the average is calculated based on monthly closing stock prices) with a strike of 100. $K = 100 \Rightarrow$ PAYOFF : $(A(T)-K)_+$

(ii) An up-and-out call option with a barrier of 125 and a strike of 120. $H_1 = 125$ $K_1 = 120$ } PAYOFF : 0

(iii) An up-and-in call option with a barrier of 120 and a strike of 110. $H_2 = 120$ $\longrightarrow$ KNOCKED IN $K_2 = 110 \Rightarrow$ PAYOFF : $115 - 110 = 5$

Calculate the difference in payoffs between the option with the largest payoff and the option with the smallest payoff.

(A) 5

(B) 10

(C) 15

(D) 20

(E) 25

$$A(T) = 100 + \frac{1}{12}\Big(+5 + 20 + 15 + 10 + 15 + \cancel{10} + 0 + \cancel{(-10)} + 5 + 25 + 10 + 15\Big)$$

$$A(T) = 100 + \frac{120}{12} = 100 + 10 \Rightarrow \text{PAYOFF} : 10$$

$\mathbb{I}_{[M(T)<H]}$

SAMPLE MFE

42. Prices for 6-month 60-strike European up-and-out call options on a stock S are available. Below is a table of option prices with respect to various H, the level of the barrier. Here, $S(0) = 50$.

H	Price of up-and-out call
60	0
70	0.1294
80	0.7583
90	1.6616
∞	4.0861

Q: Why is the price for H=60 exactly 0? K=60 => $\boxed{\text{PAYOFF} \equiv 0}$

Q: What does it mean to set H=∞? A "regular" call option.

Q: Why do the prices increase w/ respect to H? It is less likely to get KNOCKED OUT.

Consider a special 6-month 60-strike European "knock-in, partial knock-out" call option that knocks in at $H_1 = 70$, and "partially" knocks out at $H_2 = 80$. The strike price of the option is 60. The following table summarizes the payoff at the exercise date:

H_1 Not Hit	H_1 Hit	
	H_2 Not Hit	H_2 Hit
0	$2 \times \max[S(0.5) - 60, 0]$	$\max[S(0.5) - 60, 0]$

Calculate the price of the option. Our only method is to construct a REPLICATING PORTFOLIO consisting of the barrier options whose prices are given.

(A) 0.6289

(B) 1.3872

(C) 2.1455

(D) 4.5856

(E) It cannot be determined from the information given above.

$V_{SO}(T)$ ... the payoff of the special option

$V_C(T) = (S(T)-K)_+$

$$V_{SO}(T) = \underbrace{\mathbb{I}_{[M(T) \geq H_1]}}_{}\left(V_C(T) + V_C(T)\,\mathbb{I}_{[M(T) < H_2]} \right)$$

$$V_{SO}(T) = \left(1 - \mathbb{I}_{[M(T) < H_1]}\right)\left(V_C(T) + V_C(T)\,\mathbb{I}_{[M(T) < H_2]} \right)$$

Remaining steps:
- distribute (FOIL)
- idenitify and recognize the payoffs of the up-and-out calls
- calculate the price