Warm-up Worksheet #1
More on basic hedging: forward contracts

In preparation for the next class, please solve the following problems:

1.1. Payoff and profit curves for a producer of goods who hedges using a forward contract.

Problem 1.1. (2 points) Farmer Brown plans to harvest 10,000 bushels of corn. His aggregate fixed and variable costs are projected to amount to $2.47 per bushel.

At harvest time, the price of corn is observed to be $3.40 per bushel. What is farmer Brown’s total profit?

\[
\text{Total Profit} = \text{Number of Units} \times (\text{Price of Good} - \text{Cost of Production})
\]

\[
10,000 \times (3.40 - 2.47) = 10,000 \times 0.93 = 9,300.
\]

Problem 1.2. Farmer Black plans to harvest 5,000 bushels of corn. His aggregate fixed and variable costs amount to $2.75 per bushel.

(1 point) Farmer Black fears that the price of corn is going to drop, so he decides to hedge using a forward contract. Is farmer Black going to take a long or a short position in the forward contract?

\[
\text{Inherent position} : \text{Long w/ respect to "corn"} \Rightarrow \text{Hedge} = \text{SHORT FORWARD CONTRACT, i.e., he is selling forward}
\]

A forward contract on corn with delivery date at harvest time and forward price of $2.80 is available.

(2 points) At harvest time, the price of corn is observed to be $2.40 per bushel. What is the total profit of farmer Black’s hedged position?

\[
\text{Profit} = \text{Number of Units} \times (\text{Forward Price} - \text{Cost of Production})
\]

\[
5,000 \times (2.80 - 2.75) = 5000 \times 0.05 = 250 \text{ gain}
\]
(2 points) What would farmer Black's profit have been had he not decided to hedge using the forward contract?

\[ \text{# of units} \times (S(T) - C) = 5,000 \times (2.40 - 2.75) = -1,750 \]

\[ \Rightarrow 1,750 \text{ loss} \]

Problem 1.3. (2 points) Draw the payoff and profit curve (per unit) for a producer of goods who hedges using a forward contract.
Problem 1.4. (5 points) Consider the general case in which

- \( C \) stands for the total aggregate fixed and variable costs of production per unit of good;
- \( F \) stands for the forward price per unit of good.

What is the price \( s^* \) per unit of good at which the profit of a producer who hedges using a forward contract equals the profit of the producer who does not hedge at all?

\[
\begin{align*}
\text{hedged position profit} & : F - C \\
\text{unhedged profit} & : s - C
\end{align*}
\]

\[
\Rightarrow \text{Solve for } s \text{ in } F - C = s - C
\]

\[
\Rightarrow s^* = F
\]

1.2. Payoff and profit curves for a user/buyer of goods who hedges using a forward contract.

Problem 1.5. Pancakes, Inc. produces a variety of pancake products. It longed a forward contract on 100 lbs of blueberries at $2.00 per pound. Revenue is $1,000 for the pancakes produced with the above blueberries. Other costs total $700. Find the profit.

Course Website: Link to a story on "coffee prices".
$1,000 - $700 - 100 \times \$2 = \\
= \$100.

**THE GENERAL PRINCIPLE**

Unhedged Position + Hedge = Hedged position

Here:

\[ R - C - S(T) + (SCT - F) = R - C - F \]

**PROFIT**

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Diagram:
- Long forward (the hedge)
- The hedged position
- Unhedged
European call Option

NO ACTIVITY BETWEEN 0 AND T

0 \quad T \quad \text{EXERCISE DATE (also expiration or maturity date)}

• the call option is agreed upon

• the logistics:
  • the underlying asset
  • strike price $K$
  • cash/physical settlement

\rightarrow • The call price is paid!

At time-$T$: The owner of the call (i.e., the long position) has the RIGHT but NOT an obligation to buy 1 unit of an asset (i.e., the underlying) for a (predetermined) price $K$. The strike/exercise price $K$

If the purchase happens, we say that the option was EXERCISED.

Assume, all agents (e.g., the owner of the call) are behaving rationally (in the sense of profit maximization).

Q: What is the optimal behavior at time-$T$?

• Observe $S(T)$ ... the market price of underlying

• IF $S(T) \geq K$ \quad \Rightarrow \quad \text{EXERCISE} : \text{get } S(T) \text{ and give up } K

• IF $S(T) < K$ \quad \Rightarrow \quad \text{nothing} : 0
The payoff of a long call is:

\[ V_c(T) = \begin{cases} 
S(T) - K, & \text{if } S(T) \geq K \\
0, & \text{if } S(T) < K 
\end{cases} \]

Indicator random variables

\[ I_A(\omega) = \begin{cases} 
1 & \text{if } \omega \in A \\
0 & \text{if } \omega \notin A 
\end{cases} \]

\( A \subseteq \Omega \) an event

\[ V_c(T) = (S(T) - K)I_{[S(T) \geq K]} \]

Maximum operation: \( \max(a, b) = a \vee b \)

\[ \max(a, b) = \begin{cases} 
a, & \text{if } a \geq b \\
b, & \text{if } b > a 
\end{cases} \]

\[ V_c(T) = \max(S(T) - K, 0) = (S(T) - K) \vee 0 \]

The positive-part function:

\[ x \mapsto (x)_+ := \max(x, 0) \]

\[ V_c(T) = (S(T) - K)_+ \]

\[ \Rightarrow \text{The payoff function of the long call is:} \]

\[ V_c(s) = (s - K)_+ \]
Long-call payoff: $\rightarrow$ LONG

w/ respect to the underlying $\rightarrow$ not bounded from above i.e., it has the "unlimited growth potential $\rightarrow$ NONNEGATIVE".

"writer" of the option is BOUND by the option-owner's choice. Aka: SELLER, one w/ SHORT option.

NOTE: The payoff is NON-POSITIVE?

Q: Why write a call option in first place?

$\rightarrow$ There needs to be an initial PREMIUM to be paid to the writer.

$V_c(0)$... the initial value/price/premium/worth of our call option

$\Rightarrow$ PROFIT (Long Call) = PAYOFF (Long Call) - $FV_{0,T}(V_c(0))$

$= (S(T) - K)_+ - FV_{0,T}(V_c(0))$

Profit

Long-call profit

the break-even point: $K + FV_{0,T}(V_c(0))$
Q: Could a [producer/seller] of goods hedge using a [call option]?

Inherent [LONG] position.

They [SHORT/ WRITE] the call.

- [PROFIT]
  - [FV (V_{col})]
  - [-C + FV (V_{col})]
  - [-C]

- [unhedged profit]
- [hedged profit]
- [short-call profit]
- [hedge (a short call) payoff]

"A portion of the potential profit is sold by writing your call."

Also, means of financing.
Problem 3.3. The market price of the good is the independent argument \( s \). Assume that its producer is hedging using a European call. Draw the profit curve of the hedged portfolio.

Problem 3.4. Sweet potato call options with the exercise date in six months (at harvest time) and a $15.00 strike price (per carton) are trading for a $1.50 premium. Farmer Brown decides to hedge his 10,000 cartons of sweet potatoes by writing 10,000 of the above call options. The total fixed costs of producing his entire sweet-potato crop is $120,000. Assume the continuously compounded risk-free interest rate equal to 0.04. What is farmer Brown’s profit if the market price of sweet potatoes turns out to be $14.50 per carton at harvest time?

\[
\frac{S(T) - C - (S(T) - K)_+}{\text{unhedged profit}} + FV_{0,T} (V_C(0)) = \left \lfloor \begin{array}{c}
\text{short-call payoff} \\
\text{no exercise}
\end{array} \right \rfloor
\]

\[
\Rightarrow \text{Total Profit} = 10,000 \left( 14.50 + 1.50 \frac{K}{T} \right) - 120,000 =
\]

3.2. Buyer hedging with a European call.

Problem 3.5. Recall that a buyer of commodity has an inherent short position in that asset. If (s)he decides to use European calls to hedge, should (s)he buy or write the call option?