A European call gives its owner the right but not the obligation to buy the underlying asset for the (predetermined) strike price \( K \) on the exercise date \( T \).

Payoff (long call): \( V_c(T) = (S(T) - K)_+ \)

Payoff function: \( v_c(s) = (s - K)_+ \)

**Long Call**: This is a long position w/ respect to the underlying.
3.1. **Producer hedging with a European call.**

**Problem 3.1.** Recall that a producer of goods has an inherent **long position in that asset.** If (s)he decides to use European calls to hedge, should (s)he buy or write the call option?

\[ \downarrow \]

**Short/Write the call option**

**Problem 3.2.** The market price of the good is the independent argument \( s \). Assume that its producer is hedging using a European call. Draw the payoff curve of the hedged portfolio.

\[ \downarrow \]

**At home.**
Problem 3.3. PROFIT curve for a producer who hedges w/ a written call

\[ \text{FV}(V_c(0)) \]
\[ -C + \text{FV}(V_c) \]
\[ -C \]

unhedged position
hedged position
hedge (short call)

Problem 3.4.

six-month, $15$-strike calls w/ \( V_c(0) = 1.50 \)
\( C = 12 \)
\( r = 0.04 \)
\( S(T) = 14.50 \)

without hedging: \( 10,000(S(T) - C) \)

\( Q: \) Does the farmer buy or write the calls?

hedged position:

\[ 10,000 \left( S(T) - C - (S(T) - K)_+ + \text{FV}(V_c(0)) \right) = ? \]

Option is not exercised @ \( 14.50 = S(T) \)

answer = \( 10,000 \left( 14.50 - 12 + 1.50e^{0.04 \cdot 0.5} \right) = 40,303 \)
Problem 3.6. Suppose that the buyer is hedging using a European call. Draw the profit curve of the hedged portfolio.

Problem 3.7. The “Babkas, Brownies and Beyond” bakery sells blueberry muffins for $3.00 per muffin. The bakery will need to buy 100 lbs of blueberries in six months to produce the 1600 muffins needed for the “Greater Springfield Blueberry Jamboree”. Non-blueberry costs total $2,500. Assume that the continuously compounded risk-free interest rate equals 0.04. Local farmers are financially sophisticated. Our bakery uses one hundred $1.60 strike, six-month call options (each on a pound of blueberries) to hedge against rising prices of blueberries. The calls can be bought for $0.15 per call. Assume that the market price of a pound of blueberries is $1.65 in six months. What is the profit of the bakery’s hedged portfolio?

Method I: "Flat" portion of the above profit curve.

\[ R - C - FV(V_c(0)) - K = \]

\[ = 3 \cdot 1,600 - 2,500 - 100 \cdot 0.15 \cdot e^{0.04 \cdot 0.5} - 100 \cdot 1.60 \]

\[ = 2,124.70 \]

Method II: "Financial Interpretation"

\[
\text{unhedged: } R - C - S(T) \\
\text{hedge = Long Call: } (S(T) - K)^+ - FV_{0,T}(V_c(0))
\]

Profit (hedged position):

\[ R - C - \min(S(T), K) - FV_{0,T}(V_c(0)) = \]

INSTRUCTOR: Milica Čudina
In general: \(-a + (a-b)_+ = \begin{cases} -a + (a-b) & a \geq b \\ -a + 0 & b > a \end{cases} = -\min(a,b)\)
3.2.1. The short seller's perspective.

Problem 3.8. Look at the following situation:
- the initial stock price is \( S(0) = 100 \); the short seller receives the proceeds of the short sale at time \(-0\);
- the continuously compounded risk-free interest rate equals 0.04;
- the short sale is closed at time \( T = 1 \);
- a 100–strike, one-year European call on the stock is sold for $5 at time \(-0\).

What are the short seller's payoff and profit curves?

Initially: The short-seller gets \( S(0) = 100 \), to buy the call spend \( V_c(0) = 5 \)

Inflow of \( 95 \) accumulates to \( 95e^{0.04} \)
University of Texas at Austin

Quiz #6

European call options.

Please, provide the complete solution to the following problem(s):

**Problem 6.1.** The premium on a 1000-strike, 2-month European call option on the market index is $20. After 2 months the market index spot price is 1075. If the risk-free interest rate equals 0.5% effective per month, what is the long-call profit?

**Problem 6.2.** The fair price today of a zero-coupon bond with redemption amount of $100 and which comes to maturity in a year is equal to $78.

You purchase an at-the-money European call option on a non-dividend paying stock whose price today is $S(0) = $100. The premium of this call was $10.

Write the expression for this call’s payoff, and for its profit (valued at its expiration date $T$) as a function of $S(T)$ (the stock price at time $T$) and the time of maturity $T$. Draw the graph of this call’s profit as a function of $S(T)$.

**Problem 6.3.** An investor purchases a call option with an exercise price of $55 for $2.60. The same investor sells a call on the same asset with an exercise price of $60 for $1.40. At expiration, 3 months later, the asset price is $56.75. All other things being equal and given a continuously compounded annual interest rate of 4.0%, what is the profit to the investor?

**Problem 6.4.** In a certain market, you are given that

- the price of a 40–strike European call option on an underlying asset $S$ with maturity $T$ is $11;
- the price of a 50–strike European call option on an underlying asset $S$ with maturity $T$ is $6;
- the price of a 55–strike European call option on an underlying asset $S$ with maturity $T$ is $3.$

Let the risk-free interest rate be $r = 0.05.$

A trader decides to construct the following portfolio:

1. long one 40–strike call option;
2. short three 50–strike European call options;
3. long two 55–strike calls.

Suppose that at time $T = 1$ the value of the asset $S$ is $S(1) = 52.$ What is the profit of the portfolio at time $T$?

**Problem 6.5.** For what values of the final asset price is the profit of a long forward contract with the forward price $F = 100$ and delivery date $T$ in one year smaller than the profit of a long call on the same underlying asset with the strike price $K = 100$ and the exercise date $T$. Assume that the call’s premium equals $10$ and that the annual effective interest rate equals 10%.

Express your answer as an interval.

Instructor: Milica Ćudina
Profit (Long Call) = Payoff (Long Call) - FV_{0,T}(V_c(0))
= (S(T) - K)^+ - V_c(0) \cdot (1 + 0.005)^2

Answer: \((1075 - 1000)^+ - 20 (1.005)^2 = 54.80\)

\[\text{at-the-money} \Rightarrow \text{strike price} = \text{initial stock price} \quad K = S(0) = 100\]

\[V_c(0) = 10\]

Payoff: \[V_c(T) = (S(T) - 100)^+\]
Profit: \[(S(T) - 100)^+ - FV_{0,T}(10) = (S(T) - 100)^+ - \left(\frac{100}{78}\right)^T \cdot 10\]
Payoff: 
\[(S(T) - 55) + - (S(T) - 60)\]

Profit: 
\[(S(T) - 55) + - 1.20 e^{0.01} q = 1.75 - 1.20 e^{0.01}\]

Long K-strike call
Short K-strike call

Payoff curve
Long w/ respect to the underlying.