UNIVERSITY OF TEXAS AT AUSTIN

Problem set #5

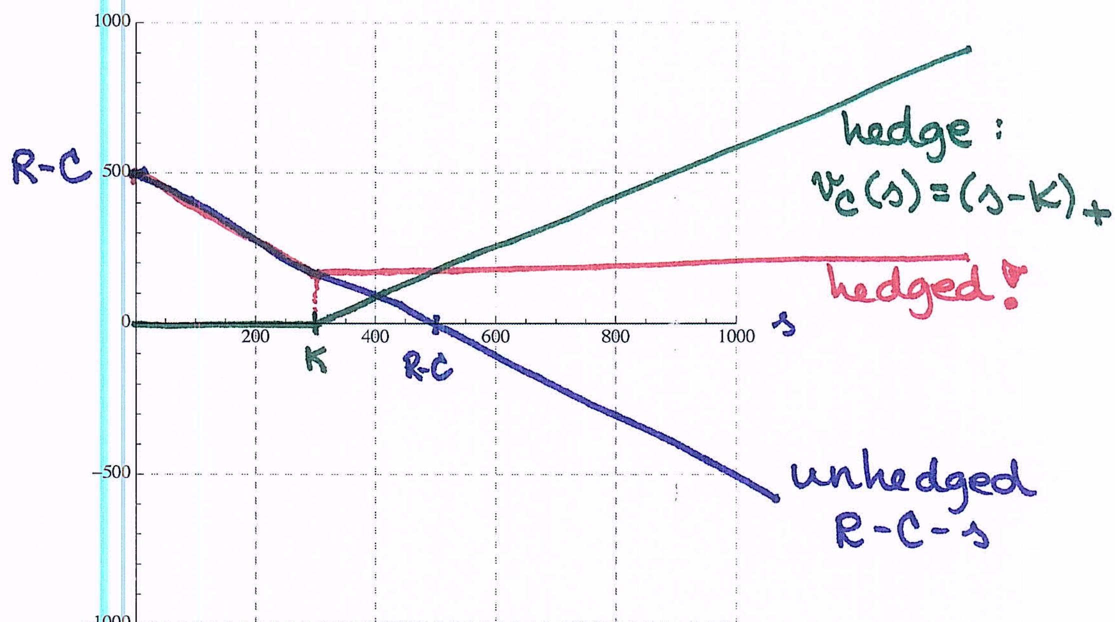
Hedging using European call options: Caps.

5.1. Buyer hedging with a European call.

Problem 5.1. Recall that a buyer of commodity has an inherent **short** position in that asset. If (s)he decides to use European calls to hedge, should (s)he buy or write the call option?

Long/Buy the call.

Problem 5.2. Suppose that the buyer is hedging using a European call. Draw the payoff curve of the hedged portfolio.



1.

Problem 5.3. The "Babkas, Brownies and Beyond" bakery sells blueberry muffins for \$3.00 per muffin. The bakery will need to buy 100 lbs of blueberries in six months to produce the 1600 muffins needed for the "Greater Springfield Blueberry Jamboree". Non-blueberry costs total \$2,500. Assume that the continuously compounded risk-free interest rate equals 0.04. Local farmers are financially sophisticated. Our bakery uses one hundred \$1.60-strike, six-month call options (each on a pound of blueberries) to hedge against rising prices of blueberries. The calls can be bought for \$0.15 per call. Assume that the market price of a pound of blueberries is \$1.65 in six months. What is the profit of the bakery's hedged portfolio?

Init. Cost: Since all other costs are valued @ time $\frac{1}{2}$, the only contribution to the initial cost is the cost of hedging, i.e., $100 \cdot 0.15 = 15$.

At t=T: $\underbrace{3 \cdot 1600}_{R} - \underbrace{2500}_{C} - \underbrace{100 \cdot S(T)}_{\text{the purchase of the underlying}} + \overbrace{100(S(T)-K)_+}^{\text{the hedge}} =$

$[K = 1.60 \; ; \; S(T) = 1.65]$

$= 4800 - 2500 - 100(1.65) + 100(\overbrace{1.65 - 1.60}^{>0})^*$

$= 2300 - 160 = 2140$

$\Rightarrow$ Profit $= 2140 - 15e^{0.04(0.5)} = 2124.70$ ∎

2.

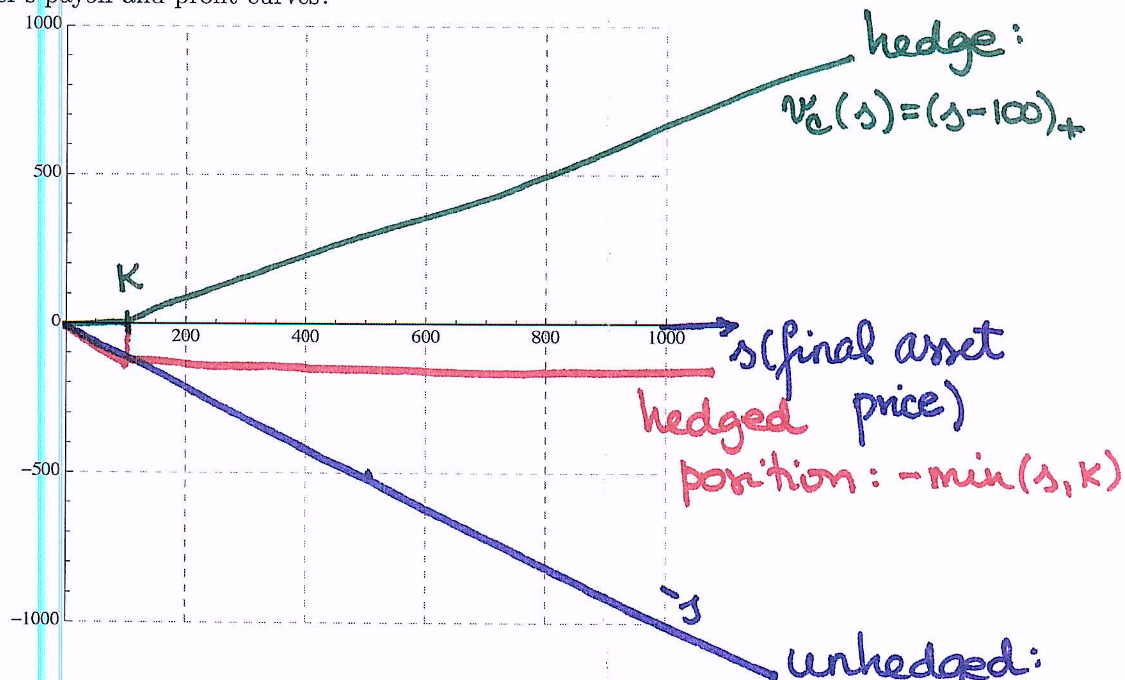
5.1.1. *The short seller's perspective.*

Problem 5.4. Look at the following situation:

- the initial stock price is $S(0) = 100$; the short seller receives the proceeds of the short sale at time-0;
- the continuously compounded risk-free interest rate equals 0.04;
- the short sale is closed at time $T = 1$;
- a 100$-$strike, one-year European call on the stock is sold for \$5 at time-0.

What are the short seller's payoff and profit curves?

Payoff



Portfolio: $\begin{cases} \bullet \text{ short stock (no dividends; for simplicity)} \\ \bullet \text{ long call} \end{cases}$ CAP

Initial cost: $\underbrace{-S(0)}_{\text{short sale}} + \underbrace{\overbrace{V_c(0)}^{=5}}_{\text{long call}}$

Payoff: $\underbrace{-S(T)}_{} + \underbrace{(S(T)-K)_+}_{} =$

$$= \begin{cases} -K, & \text{IF } S(T) \geq K \\ -S(T), & \text{IF } S(T) < K \end{cases}$$

$= -\text{MIN}(S(T), K)$

Covered / Naked Option Writing

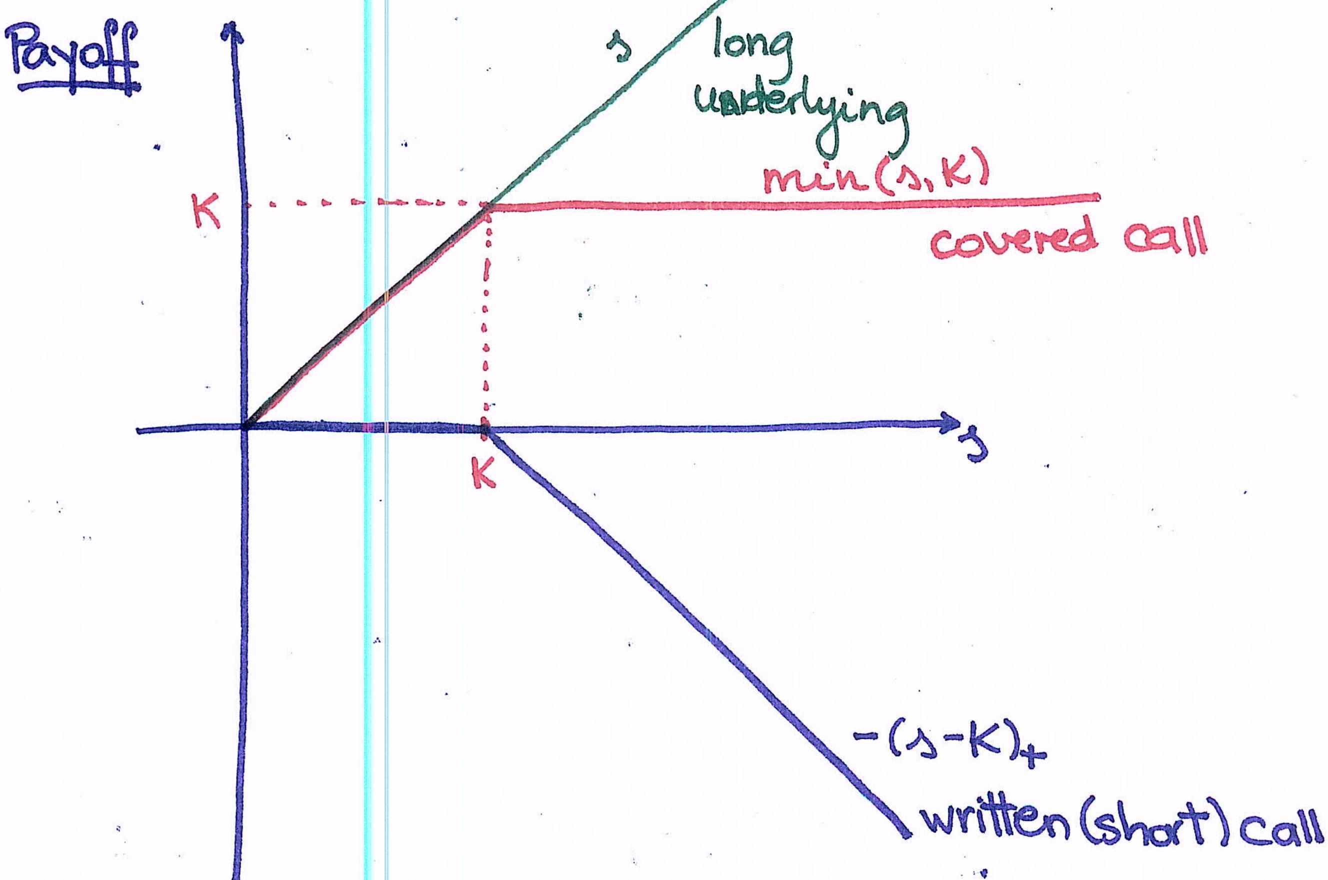
Covered ... if the option's writer has a simultaneous appropriate (opposite) position in the underlying asset

Naked ... if not covered, i.e., no opposite position in the underlying

e.g., w/ a call: (for simplicity, assume no dividends)

- written call option
- long/buy the underlying

$\Big\}$ COVERED CALL



47.

An investor has written a covered call.

Determine which of the following represents the investor's position.

(A) Short the call and short the stock wrong direction

(B) Short the call and long the stock

(C) Short the call and no position on the stock naked writing

(D) Long the call and short the stock

(E) Long the call and long the stock

42.

An investor purchases a non-dividend-paying stock and writes a t-year, European call option for this stock, with call premium C. The stock price at time of purchase and strike price are both K. $S(0) = K$

Assume that there are no transaction costs.

The risk-free annual force of interest is a constant r. Let S represent the stock price at time t.

$S > K$. $\Rightarrow$ The call will be exercised

Determine an algebraic expression for the investor's profit at expiration.

(A) Ce^{rt}

(B) $C(1+rt) - S + K$

(C) $Ce^{rt} - S + K$

(D) $Ce^{rt} + K\left(1 - e^{rt}\right)$

(E) $C(1+r)^t + K\left[1 - (1+r)^t\right]$

Portfolio: $\boxed{\text{covered call}}$

using the notation above

Initial Cost: $S(0) - V_c(0) = K - C$

Payoff: $\min(S(T), K) = K$

$S(T) > K$

$\Rightarrow$ Profit = Payoff − $FV_{0,t}$(Initial cost)

$= K - (K - C)e^{r \cdot t} = \underbrace{K(1 - e^{r \cdot t}) + Ce^{r \cdot t}}_{\Downarrow\ (D)}$

6.