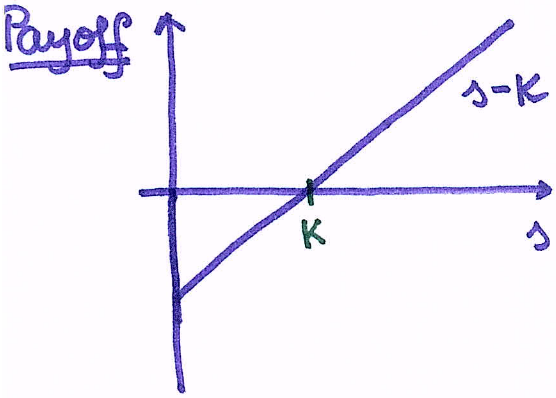
Put-Call Parity.

Portfolio A:
- long call
- short put

} both European & otherwise identical



The payoff:

$$V_A(T) = V_C(T) - V_P(T) = S(T) - K$$

Portfolio B:
- long prepaid forward w/ delivery date T
- borrow $PV_{0,T}(K)$ @ the risk-free rate to be repaid @ time-T

$$\Rightarrow V_B(T) = S(T) - K$$

$\Rightarrow$ for any $t \in [0,T]$: $V_A(t) = V_B(t)$

$$V_C(t) - V_P(t) = F^P_{t,T}(S) - PV_{t,T}(K)$$

In particular, for $t=0$:

$$V_C(0) - V_P(0) = F^P_{0,T}(S) - PV_{0,T}(K)$$

PUT-CALL PARITY.

Note:
- The no-arbitrage assumption is sufficient to get put-call parity.
- Only works for European options.

1. Consider a European call option and a European put option on a nondividend-paying stock. You are given:

(i) The current price of the stock is 60. $S(0) = 60$

(ii) The call option currently sells for 0.15 more than the put option. $V_c(0) - V_p(0) = 0.15$

(iii) Both the call option and put option will expire in 4 years. $T = 4$

(iv) Both the call option and put option have a strike price of 70. $K = 70$

Calculate the continuously compounded risk-free interest rate. $r = ?$

(A) 0.039

(B) 0.049

(C) 0.059

(D) 0.069

(E) 0.079

Put·Call Parity:

$$\underbrace{V_c(0) - V_p(0)}_{0.15} = \underbrace{F^P_{0,T}(S)}_{\underbrace{S(0)}_{\text{no dividends}}} - Ke^{-rT}$$

$$0.15 = S(0) - Ke^{-rT}$$

$$= 60 - 70e^{-4r}$$

$$\Rightarrow 70e^{-4r} = 60 - 0.15 = 59.85$$

$$\Rightarrow -4r = \ln\left(\frac{59.85}{70}\right)$$

$$\Rightarrow r = -\frac{1}{4}\ln\left(\frac{59.85}{70}\right) = \ldots = 0.039 \blacksquare$$

Note: • The r solved for in the put·call parity is sometimes called the IMPLIED INTEREST RATE.

2.

BEGINNING OF EXAMINATION
ACTUARIAL MODELS – FINANCIAL ECONOMICS SEGMENT

1. On April 30, 2007, a common stock is priced at $52.00. You are given the following:

(i) Dividends of equal amounts will be paid on June 30, 2007 and September 30, 2007.

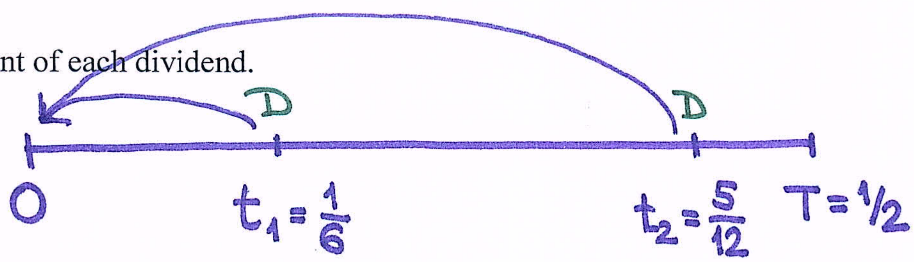
D… dividend amt

(ii) A European call option on the stock with strike price of $50.00 expiring in six months sells for $4.50. $= V_c(0)$ $K = 50$

(iii) A European put option on the stock with strike price of $50.00 expiring in six months sells for $2.45. $= V_p(0)$

(iv) The continuously compounded risk-free interest rate is 6%. $r = 0.06$

Calculate the amount of each dividend.



(A) $0.51

(B) $0.73

(C) $1.01

(D) $1.23

(E) $1.45

$S(0) = 52$

Put·Call Parity:

$$V_c(0) - V_p(0) = \underbrace{F^P_{0,T}(S)}_{= S(0) - PV(\text{Dividends})} - PV_{0,T}(K)$$

$$\Rightarrow 4.50 - 2.45 = 52 - D\left(e^{-\frac{1}{6}\cdot 0.06} + e^{-\frac{5}{12}\cdot 0.06}\right) - 50e^{-\frac{1}{2}\cdot 0.06}$$

$$\Rightarrow D = \frac{52 - 50e^{-0.03} - 4.50 + 2.45}{e^{-0.01} + e^{-0.025}} = \dots = 0.73 \Rightarrow \text{B.}$$

53. $F_{0,4} = 300$

For each ton of a certain type of rice commodity, the four-year forward price is 300. A four-year 400-strike European call option costs 110. $V_c(0) = 110$

$T=4$ $K=400$

The continuously compounded risk-free interest rate is 6.5%.

$r = 0.065$

Calculate the cost of a four-year 400-strike European put option for this rice commodity.

Put·call Parity:

$$V_c(0) - V_p(0) = F^P_{0,T} - PV_{0,T}(K)$$
$$= PV_{0,T}(F_{0,T} - K)$$
$$= (F_{0,T} - K)e^{-rT}$$

r...ccrfir

(A) 10.00

(B) 32.89

(C) 118.42

(D) 187.11

(E) 210.00

54.

DELETED

$$V_p(0) = V_c(0) + e^{-rT}(K - F_{0,T})$$
$$= 110 + e^{-0.065 \cdot 4}(400 - 300) = \dots = 187.11 \blacksquare$$

55.

Box spreads are used to guarantee a fixed cash flow in the future. Thus, they are purely a means of borrowing or lending money, and have no stock price risk.

Consider a box spread based on two distinct strike prices (K, L) that is used to lend money, so that there is a positive cost to this transaction up front, but a guaranteed positive payoff at expiration.

Determine which of the following sets of transactions is equivalent to this type of box spread.

(A) A long position in a (K, L) bull spread using calls and a long position in a (K, L) bear spread using puts.

(B) A long position in a (K, L) bull spread using calls and a short position in a (K, L) bear spread using puts.

(C) A long position in a (K, L) bull spread using calls and a long position in a (K, L) bull spread using puts.

(D) A short position in a (K, L) bull spread using calls and a short position in a (K, L) bear spread using puts.

(E) A short position in a (K, L) bull spread using calls and a short position in a(K, L) bull spread using puts.

4.

If $K = F_{0,T}$, $\boxed{\text{long call + short put}}$

is a SYNTHETIC FORWARD;

not surprisingly $V_C(0) = V_P(0)$

Frequently, we say:

- long call + short put

is a synthetic forward even when $K \neq F_{0,T}$.

The forward contract:

$F_{0,T}(S) = FV_{0,T}(S(0))$

$= 1{,}000(1+i)^T$

$= 1{,}050$

5.

The PS index has the following characteristics:

- One share of the PS index currently sells for 1,000.
- The PS index does not pay dividends.

$\Rightarrow$ Must spend money up front.

Sam wants to lock in the ability to buy this index in one year for a price of 1,025. He can do this by buying or selling European put and call options with a strike price of $\underbrace{1{,}025}_{K}$.

The annual effective risk-free interest rate is 5%. $i = 0.05$

Determine which of the following gives the hedging strategy that will achieve Sam's objective and also gives the cost today of establishing this position.

$\Downarrow$ ~~A~~, ~~B~~, ~~C~~

(A) ~~Buy the put and sell the call, receive 23.81~~

(B) ~~Buy the put and sell the call~~, spend 23.81

(C) ~~Buy the put and sell the call, no cost~~

(D) Buy the call and sell the put, ~~receive 23.81~~

(E) Buy the call and sell the put, spend 23.81

$F_{0,T} - K$

$23.81 = PV_{0,1}(25) = \frac{25}{1.05}$

6.

The following relates to one share of XYZ stock:

- The current price is 100.
- The forward price for delivery in one year is 105.
- P is the expected price in one year

Determine which of the following statements about P is TRUE.

(A) $P < 100$

(B) $P = 100$

(C) $100 < P < 105$

(D) $P = 105$

(E) $P > 105$

Q: What if put-call parity does not hold for certain observed prices?

$\Rightarrow$ There is an arbitrage opportunity!

Example. Continuous dividends.

Assume: $V_c(0) - V_p(0) \neq F^P_{0,T}(S) - PV_{0,T}(K)$

Without loss of generality:

$$\underbrace{V_c(0) - V_p(0)}_{\text{relatively cheap.}} < \underbrace{S(0)e^{-\delta \cdot T} - PV_{0,T}(K)}_{\text{relatively expensive}}$$

- long the call
- write the put
- short sell $e^{-\delta T}$ shares

Init. Cost: $V_c(0) - V_p(0) - e^{-\delta \cdot T} \cdot S(0) < -PV_{0,T}(K)$

Payoff: $\underbrace{(S(T)-K)_+ - (K-S(T))_+}_{\cancel{S(T)} - K} - S(T) =$

$$\cancel{S(T)} - K \quad - \cancel{S(T)} = -K$$

Profit: $> -K + FV_{0,T}(PV_{0,T}(K)) = 0$

$\Rightarrow$ Arbitrage!

Q: What if put-call parity does not hold for some (certain) observed prices?

$\Rightarrow$ there will be an arbitrage opportunity!

Example. Continuous-dividends.

Assume: $V_C(0) - V_P(0) \neq F^P_{0,T}(S) - PV_{0,T}(K)$

Without loss of generality:

$$\underbrace{V_C(0) - V_P(0)}_{\text{relatively expensive}} > \underbrace{F^P_{0,T}(S) - PV_{0,T}(K)}_{\text{relatively cheap.}} \quad \star$$

$$F^P_{0,T}(S) = e^{-\delta \cdot T} S(0)$$

- long $e^{-\delta \cdot T}$ shares of stock
- write the call option
- buy the put option

Initial cost: $e^{-\delta \cdot T} S(0) - V_C(0) + V_P(0) \overset{\star}{<} PV_{0,T}(K)$

Payoff: $S(T) - (S(T) - K) = K$

$\Rightarrow$ Profit: $K - FV_{0,T}(\text{Init. Cost}) > \underbrace{K - FV(PV(K))}_{} = 0$

Arbitrage portfolio!