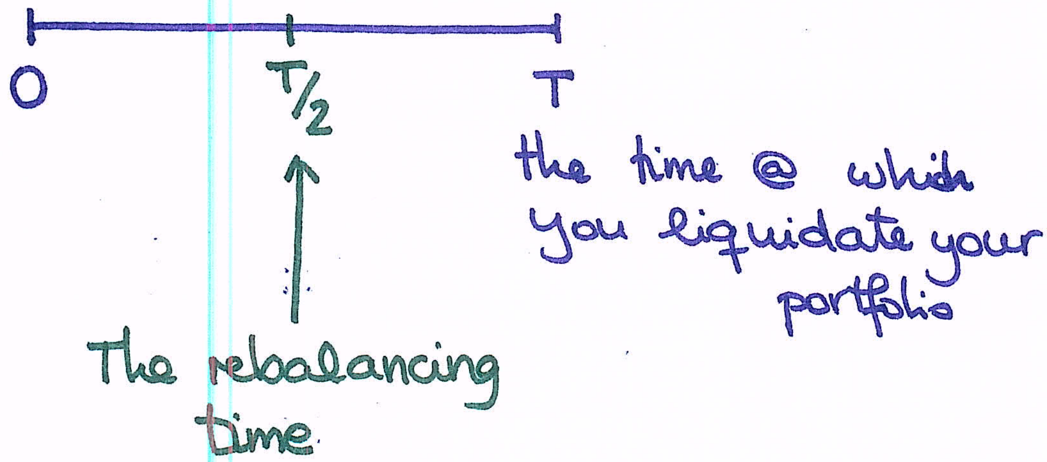
Example. You buy <u>one</u> share of non-dividend-paying stock @ time 0.



Observe $\boxed{S(T/2)}$.

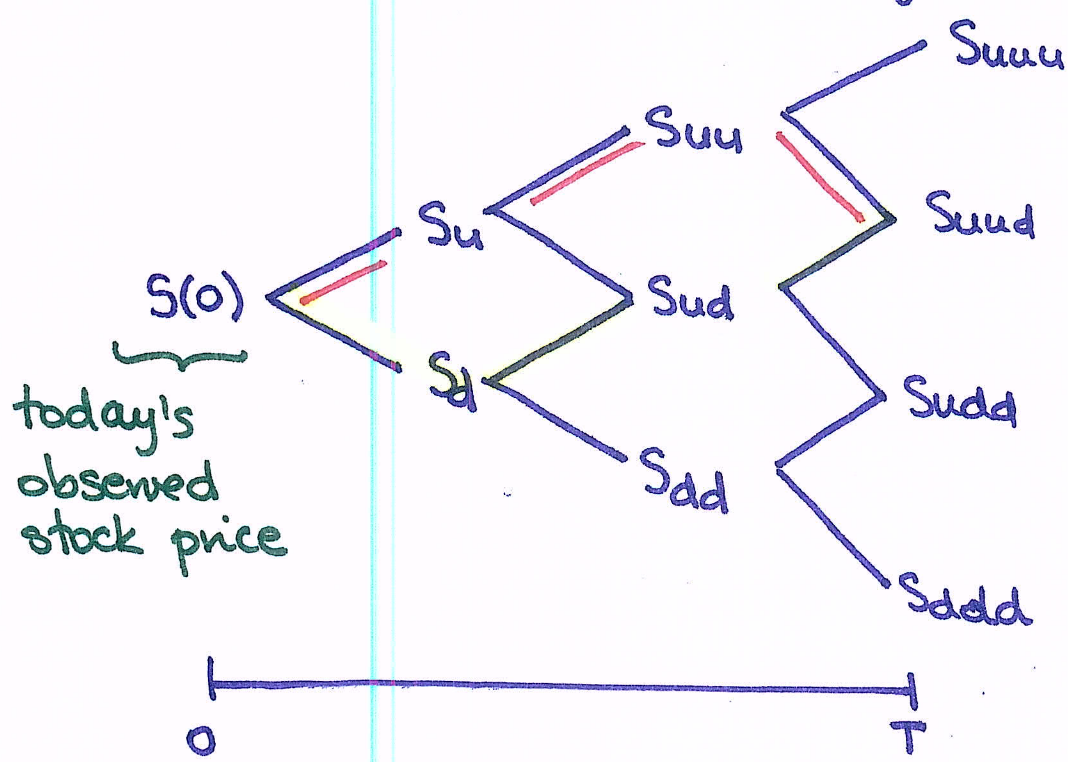
IF $S(T/2) < S(0)$, <u>then</u> buy 1 more share.

IF $S(T/2) \geq S(0)$, <u>then</u> sell $\dfrac{S(T/2)-S(0)}{S(T/2)}$ shares.

D: Feb 27th, 2019.

Finite Probability Space

... serve as our model for the possible paths of the price of the underlying asset, e.g.,



Rnd variable $S(T)$ can take only the values:
Suuu, Suud, Sddu, Sddd

All of the *finitely* many possible scenarios are called states of the world.

We assume that:

- each one can happen, i.e., its probab. > 0

and

- they all EXHAUST the outcome space, i.e., $\sum$ probab. $= 1$

Def'n. An ARBITRAGE PORTFOLIO is a portfolio whose PROFIT is:

- NON-NEGATIVE in ALL states of the world

AND

- STRICTLY POSITIVE in AT LEAST ONE state of the world

***

Unless it is specified in a particular problem/example that we are seeking an arbitrage opportunity, we assume NO ARBITRAGE!

***

Law of the Unique Price

For simplicity, focus on two static portfolios: A and B.

Assume that their payoffs are equal, i.e.,

$$V_A(T) = V_B(T)$$

3.

In general, two random variables X & Y are said to be equal if $\mathbb{P}[X=Y]=1$. On a finite probab. space, they must take the same value for every elementary outcome.

Our claim: $\boxed{V_A(0) = V_B(0)}$

→: Assume, to the contrary, that

$$V_A(0) \neq V_B(0).$$

Without loss of generality:

$$\underbrace{V_A(0)}_{\substack{\text{relatively} \\ \text{cheap}}} < \underbrace{V_B(0)}_{\substack{\text{relatively} \\ \text{expensive}}}$$

Propose an arbitrage portfolio:

- LONG Portfolio A
- SHORT Portfolio B

} Total Portfolio

Verify that this is, indeed, an arbitrage portfolio.

- Initial Cost: $V_A(0) - V_B(0) < 0$

  ⇓

  Inflow of money @ time 0

- Payoff: $V_A(T) - V_B(T) = 0$

$$\Rightarrow \quad \text{Profit} = \text{Payoff} - FV_{0,T}(\text{Init. Cost})$$
$$= -FV_{0,T}(V_A(0) - V_B(0))$$
$$> 0$$

We have created an arbitrage portfolio! ⇒⇐

Def'n. Consider a European-style derivative security. A static portfolio w/ the same payoff as the derivative security is said to be its REPLICATING PORTFOLIO.

Note: The initial price of the replicating portfolio <u>must</u> be equal to the price of the derivative security.

Review:

- Outright Purchase
- Fully-leveraged Purchase
- FORWARD CONTRACT