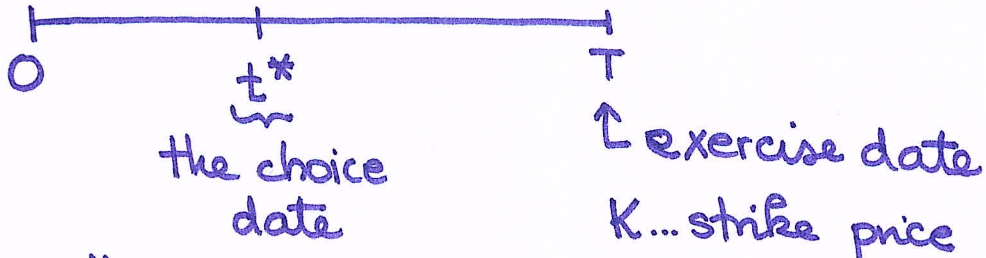
02/28/2018.

Chooser Options.



At time-t^*, the chooser option's owner decides whether the option becomes a call or a put (either with strike K and exercise @ T).

Assume the owner is rational.

Q: What criterion does the owner use @ time-t^*?

The owner compares the prices of the call & put w/ strike K & exercise @ T, and chooses the one w/ the higher price.

<u>Notation</u>:

- $V_{CH}(\underset{\substack{\uparrow\\\text{valuation date}}}{t}, \underset{\substack{\uparrow\\\text{choice date}}}{t^*}, \overset{\text{exercise date}}{T})$
- $V_{\substack{c\\p}}(t, \text{exercise date}, \text{strike price})$

$$\Rightarrow V_{CH}(t^*, t^*, T) = \max\left(V_c(t^*, T, K), V_p(t^*, T, K)\right)$$

$$\overset{a\vee b}{\overset{\shortparallel}{\max(a,b)}} = a + \underline{\max(0, b-a)} = a + (b-a)_+$$
$$= b + \max(a-b, 0) = b + (a-b)_+$$

$$\Rightarrow V_{CH}(t^*, t^*, T) = V_c(t^*, T, K) + \underbrace{\Big(V_p(t^*,T,K) - V_c(t^*,T,K)\Big)}_{\substack{\parallel \text{ Put·call Parity!} \\ PV_{t^*,T}(K) - F^P_{t^*,T}(S)}}{}_+$$

For simplicity, assume:
- no dividends
- r... ccrfir

$$\Rightarrow V_{CH}(t^*, t^*, T) = \underbrace{V_c(t^*,T,K)} + \underbrace{\Big(Ke^{-r(T-t^*)} - S(t^*)\Big)_+}_{\substack{\text{Payoff of a PUT} \\ \text{w/ strike } Ke^{-r(T-t^*)} \\ \text{and exercise date } t^*}}$$

⇒ A replicating portfolio for the chooser option:

$$\begin{cases} \bullet \text{ a } \underline{\text{long}} \text{ call w/ strike } K \text{ \& exercise date } T \\ \bullet \text{ a } \underline{\text{long}} \text{ put w/ strike } Ke^{-r(T-t^*)} \text{ \& exercise @ } t^* \end{cases}$$

$$\Rightarrow V_{CH}(0, t^*, T) = V_{\textcircled{c}}(0, T, K) + V_{\textcircled{p}}(0, t^*, Ke^{-r(T-t^*)})$$
$$= V_p(0,T,K) + V_c(0,t^*, Ke^{-r(T-t^*)})$$

25. Consider a chooser option (also known as an as-you-like-it option) on a nondividend-paying stock. At time 1, its holder will choose whether it becomes a European call option or a European put option, each of which will expire at time 3 with a strike price of \$100.

The chooser option price is \$20 at time $t = 0$.

The stock price is \$95 at time $t = 0$. Let $C(T)$ denote the price of a European call option at time $t = 0$ on the stock expiring at time T, $T > 0$, with a strike price of \$100.

You are given:

(i) The risk-free interest rate is 0.

(ii) $C(1) = \$4$.

$V_c(0,1,100) = 4 \Rightarrow V_P(0,1,100) =$

$= V_c(0,1,100)$

$+ PV_{0,1}(100) - S(0)$

$= 4 + 100 - 95 = 9$

Determine $C(3)$.

(A) \$ 9

(B) \$11

(C) \$13

(D) \$15

(E) \$17

● $V_c(0,3,100) = V_{CH}(0,1,3) - V_P(0,1,100)$

$= 20 - 9 = 11$ ■

③

3. An insurance company sells single premium deferred annuity contracts with return linked to a stock index, the time-t value of one unit of which is denoted by $S(t)$. The contracts offer a minimum guarantee return rate of $g\%$. At time 0, a single premium of amount π is paid by the policyholder, and $\pi \times y\%$ is deducted by the insurance company. Thus, at the contract maturity date, T, the insurance company will pay the policyholder

$$\pi \times (1 - y\%) \times \text{Max}[S(T)/S(0), (1 + g\%)^T].$$

You are given the following information:

(i) The contract will mature in one year. $T=1$

(ii) The minimum guarantee rate of return, $g\%$, is 3%.

(iii) Dividends are incorporated in the stock index. That is, the stock index is constructed with all stock dividends reinvested. $\delta=0$

(iv) $S(0) = 100$.

(v) The price of a one-year European put option, with strike price of $103 on the stock index is $15.21.

Determine $y\%$, so that the insurance company does not make or lose money on this contract.

The insurance company's liability is:

$$\underbrace{\Pi\,(1-y)}_{\text{const.}} \times \underbrace{\max\left[\frac{S(T)}{S(0)}, (1+g)^T\right]}$$

$$\frac{1}{S(0)}\cdot \max\left[S(T), \underbrace{\underbrace{S(0)}_{100}\cdot\underbrace{(1+g)^T}_{(1.03)^1}}_{103}\right]$$

Const.

$$\left(\underbrace{S(T)}_{\uparrow} + \underbrace{\max(0, 103 - S(T))}_{\uparrow}\right)$$

The payoff of a prepaid forward.

The payoff of the put from (v). :)

$\Rightarrow$ If $\cancel{\Pi} = \cancel{\Pi}\,(1-y)\cdot\frac{1}{100}\left(\overset{F^P_{0,1}(S)}{100} + \overset{V_P(0)}{15.21}\right)$,

<u>then</u> the insurance company breakes even.

$$\Rightarrow \quad 1-y = \frac{100}{115.21} \Rightarrow y = \frac{15.21}{115.21} \approx 0.132 \quad \blacksquare$$