UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 5

Convexity. Ratio spreads. The binomial asset-pricing model.

5.1. **Butterfly spreads and convexity.** Please, provide your **complete solution** to the following problem(s):

Problem 5.1. (5 points) In a certain market, you are given that

the price of a 40−strike European call option on an underlying asset S and with maturity T is \$11;
the price of a 50−strike European call option on an underlying asset S and with maturity T is \$6;
the price of a 55−strike European call option on an underlying asset S and with maturity T is \$3.

Let the risk-free interest rate be $r = 0.05$.

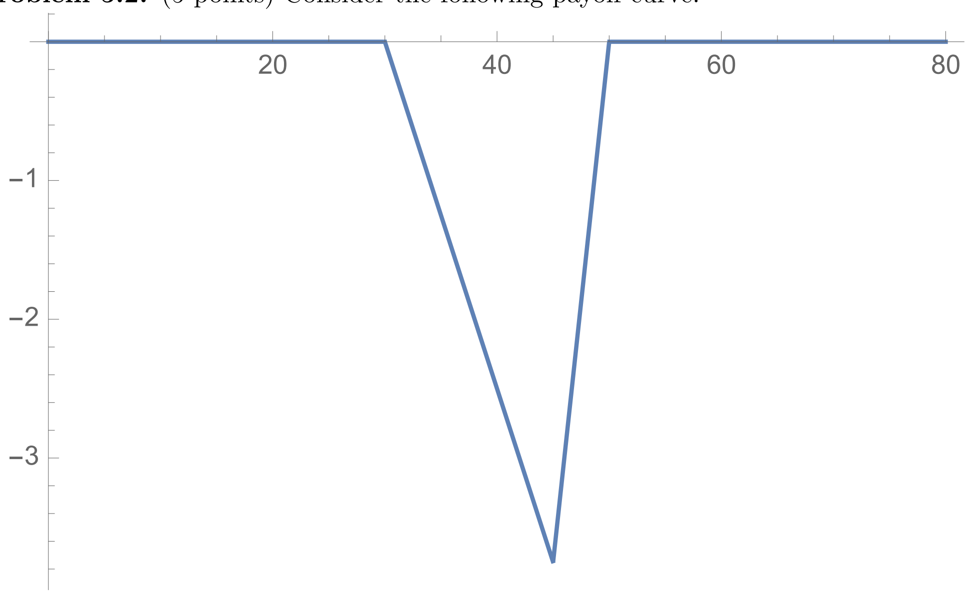
A trader decides to construct the following portfolio:

(1) long one 40−strike call option;
(2) short three 50−strike European call options;
(3) long two 55−strike calls.

Suppose that at time $T = 1$ the value of the asset S is $S(1) = 52$. What is the profit of the portfolio at time T?

Provide your **final answer** only for the following problems.

Problem 5.2. (5 points) Consider the following payoff curve:



Which of the following positions has the above payoff?

(a) A long butterfly spread.
(b) A short butterfly spread.
(c) A long strangle.
(d) A short straddle.
(e) None of the above.

Problem 5.3. (5 points) Let $K_1 = 50, K_2 = 55$ and $K_3 = 65$ be the strikes of three European call options on the same underlying asset and with the same exipration date. Let $V_C(K_i)$ denote the price at time-0 of the option with strike K_i for $i = 1, 2, 3$.
We are given that $V_C(K_1) = 16$ and $V_C(K_3) = 1$. What is the maximum possible value of $V_C(K_2)$ which still does not violate the convexity property of option prices?

(a) $V_C(K_2) = 10$
(b) $V_C(K_2) = 11$
(c) $V_C(K_2) = 13$
(d) $V_C(K_2) = 15$
(e) None of the above.

Problem 5.4. (5 points) You are interested in purchasing a European call option on the underlying asset S which has expiration date $T = 1$ year and strike $K = 70$. Denote the price of this call by C.
You are given that the premiums for European call options with the same expiration date and the same underlying asset but with the strikes $K_1 = 60$ and $K_2 = 75$ are $C_1 = 10$ and $C_2 = 3$, respectively.
You know that there is no arbitrage in your market. Find the maximal price C that you might be charged so that none of the convexity inequalities are violated.

(a) $C = 16/3$
(b) $C = 6$
(c) $C = 25/3$
(d) $C = 28/3$
(e) None of the above.

Problem 5.5. (5 points) Consider three European put options on the same stock with the same exercise date. The put premium for the $32-$strike option is 2.50 and the put premium for the $37-$strike option is 6.50. What can you say about the $40-$strike put option?

(a) Its highest possible premium is \$8.90.
(b) Its lowest possible premium is \$8.90.
(c) Its highest possible premium is \$10.50
(d) Its lowest possible premium is \$10.50.
(e) None of the above.

Problem 5.6. (5 points) Calculate the price of a long butterfly spread constructed using the following call options:

(1) a £3,925$-$strike call on the FTSE100 index which is being sold for £713.07;
(2) a £4,325$-$strike call on the FTSE100 index which is being sold for £496.46;
(3) a £4,725$-$strike call on the FTSE100 index which is being sold for £333.96.

Assume that the total number of the call options in your portfolio equals 4.

(a) £54.11
(b) £550.57
(c) £554.11
(d) £559.57
(e) None of the above

5.2. **Ratio spreads.** Please, provide your **final answer only** for the following problem(s):

Problem 5.7. (5 points) Consider the ratio spread consisting of:
- five long \$40-strike, one-year calls on **S**,
- seven short \$60-strike, one-year calls on **S**.

You model the stock price at time−1 using the following model

$$S(1) \sim \begin{cases} \$35, & \text{with probability } 0.15 \\ \$45, & \text{with probability } 0.25 \\ \$55, & \text{with probability } 0.35 \\ \$65, & \text{with probability } 0.25 \end{cases}$$

What is the expected payoff of the ratio spread above?

(a) 45
(b) 50
(c) 55
(d) 60
(e) None of the above

5.3. **Box spreads.** Please, provide your **complete solution** to the following problem(s):

Problem 5.8. (5 points) Solve Sample IFM (Derivatives: Intro) Problem **#17**.

5.4. **The binomial asset-pricing model.** Provide your **complete solution** for the following problem(s).

Problem 5.9. (10 points) Assume that one of the no-arbitrage conditions in the binomial model for pricing options on a non-dividend paying stock S is violated. Namely, let

$$e^{r \cdot h} \leq d < u.$$

Illustrate that the above inequalities indeed violate the no-arbitrage requirement. In other words, construct an arbitrage portfolio and show that your proposed arbitrage portfolio is, indeed, and arbitrage portfolio.